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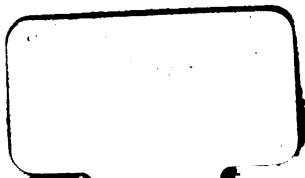
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**CHAPIN**

THE  
IMPROVED ABACUS:  
AN EXPLANATORY TREATISE  
ON THE  
THEORY AND PRACTICE  
OF  
ARITHMETIC AND MENSURATION.

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BY THOMAS RAINEY.

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ENERGY IS THE PRICE OF SUCCESS.

CINCINNATI:  
PUBLISHED BY E. D. TRUMAN  
111, MAIN STREET.

1849.

## RECOMMENDATIONS.

### *From the Cincinnati Evening Nonpareil.*

"We have received a copy of 'Rainey's Improved Abacus,' a treatise on Arithmetical calculations, and take pleasure in recommending it as a work deserving the patronage of business men desiring a knowledge of a *plain, concise, and practical system*. It simplifies and shortens calculations by canceling or expunging the numbers, and is of incalculable benefit to those whose business requires of them a *simplified method of calculation*."

### *From the Cincinnati Daily Evening Times.*

"Messrs. E. D. Truman & Co. have just issued a Treatise on Arithmetic by the canceling method. The work is by Prof. Rainey, and is well prepared for popular instruction. All questions in numbers are of either Multiplication or Division, and this is the base of the system. We can truly say, the canceling method is a superior one for all ordinary calculations, and we trust it will be generally cultivated. Mr. Rainey has given to the public his analysis in cheap form, and they are under great obligations therefor."

### *From the Cincinnati Great West.*

"THE IMPROVED ABACUS: an explanatory treatise on the Theory and Practice of Arithmetic and Mensuration, in which the general principles involved in practical calculations are thoroughly elucidated and illustrated by numerous analytic and abbreviated examples. By Thomas Rainey. Energy is the price of success."

"This is the title of a neat volume from the press of Messrs. Truman & Co. The design of the work is to give to *practical men and advanced students, a thorough theoretical and practical knowledge of all such calculations as are necessary in commercial and mechanical transactions*; and this, too, on *common sense principles*; 'for,' says the author, '*rules without reasons are ridiculous, and insulting to the inquiring mind*.' That a work of this character has long been needed, no practical man can deny; and every advanced student in the science will hail its appearance with delight."

"Knowing practically, the many deficiencies of the old systems used, we cannot too highly recommend this excellent treatise."

### *From Prof. Wm. McGOOKIN, Sidney, O.*

"I have examined a small work published by Prof. T. Rainey, on Interest, Profit and Loss, Proportions, Discounts, Insurance, Stocks, Commission, Mensuration, etc., and am much pleased with it. It is just such a work as the learner needs; it supersedes the necessity of a teacher. The rules are concise and practical."

WM. MCGOOKIN."

### *From Prof. PRATT, University of Missouri.*

"Having attended your course of lectures, in Arithmetic, etc., you will allow me to express to you my entire conviction that your system is peculiarly adapted to the simplification of Arithmetical calculations generally, and to the abbreviation of most intricate computations."

Very respectfully, &c.,

GEO. C. PRATT."

"I fully concur with Prof. PRATT."

WM. T. DAVIS,  
Columbia High School."

### *From H. F. WEST, of Indianapolis.*

\* \* \* \* \* "The unquestionable accuracy, as well as the facility of Prof. Rainey's process of solving problems, commends itself to the plain common sense of every observer, as it is so happily adapted to the practical purposes of life. . . . The ready manner by which the results of intricate questions were obtained, correctly too, showed that Prof. Rainey had happily adapted the principles of his improvements to all those perplexing rules that fill up our school Arithmetics. \* \* \* Every business man, and every young man that intends being one, should not be unacquainted with this important attainment in the science of numbers."

*From Prof. McWILLIAMS, Springfield, O., late Presiding Officer of Ohio School Convention.*

\* \* \* \* \* "He lectures at present to our school at large, and from this test, I find him to be what he professes. \* \* \* Do not fail, if practicable at all, to try his system; and the farther the better."

*From the Cincinnati Gazette.*

.... "The author commences by showing that if a quantity is to be multiplied and divided by the same number, it will remain unchanged in value; and consequently neither operation need be performed: Or, if a quantity is to be multiplied by a given number and divided by another number half as great, we may in the first instance, multiply by half the number, and neglect the division altogether, and the result will be the same that it would have been, had we performed both operations, and gone through with twice the amount of labor. The author does not confine himself to the simple examples we have given, but shows that this great principle—that opposing forces of equal strength will destroy each other—runs through the whole system, and that it may be used to immense advantage under all of its rules; in fact, it is the main pillar upon which the author has reared his beautiful system. .... What next strikes the reader of this instructive work, is, that the author permits no opportunity of imparting instruction to pass by him unimproved. The terms of art which he uses are all clearly and beautifully explained; and the mechanical and tedious methods of instruction adopted by many of the old writers, are exposed."

From the Tennessee [Nashville] Organ—Rev. JOHN P. CAMPBELL, Editor.

**✂ A NEW AND USEFUL BOOK FOR EVERYBODY. ✂**  
**RAINEY'S IMPROVED ABACUS: AN EXPLANATORY TREATISE ON THE THEORY AND PRACTICE OF ARITHMETIC AND MEASUREMENT.**

THE work above noticed, was written with direct reference to the wants of all classes of business men. The system is new, very short, and beautiful.

Experience teaches the business man that the old system requires too many figures: that the rules are too tedious and arbitrary: that in too many instances results are found by the arbitrary and incomprehensible arrangements of the books; and that if he is unable to retain or apply such rules, he must fall back on common sense, and construct his own rules. Now, all rules, without reasons, we consider, ridiculous and insulting to the intelligent and inquiring mind: but with reasons presented first, and afterward calculations to illustrate their application and use, it becomes an easy matter for any person of common mind to make his own rules. Hence, every rule in Arithmetic and Mensuration is fully, clearly, and satisfactorily explained; while a great number of examples, such as occur in everyday business, are wrought out; thus enabling the practical man to find in the book the method of making calculations similar to any that he may find necessary.

The practical calculator will find in this work a system of calculations extraordinarily short, simple, and satisfactory; and explained in PLAIN, FAMILIAR LANGUAGE, free from difficult terms, which too often obscure the sense, without satisfying the judgment.

No department of calculations is omitted in this work, which would be of interest to business men and students, of any trade or profession.

MERCHANTS and COMMERCIAL men will find everything relating to their vocations, and a great variety of new and interesting matter, not before introduced into any work of this kind. We invite particular attention to the system of Interest, Discount, Profits and Losses, the several varieties of Commission, Insurance, etc.; the combination of several different statements in one statement; and to the Tables for Banking, Equation, etc.

MECHANICS are invited to examine the great variety of work pertaining to MACHINERY; the MECHANICAL POWERS; the TABLES OF AREAS, etc.; WEIGHTS OF METALS, etc.; Circles, Cylinders, Globes, Balls, etc.; Contents and Weights of Solid Bodies, and all of the Superficial and Solid Measurements necessary in any department of mechanics.

The attention of SCIENTIFIC men, is directed to the new and beautiful system of PROPORTIONS, CAUSE and EFFECT; the Philosophy of the general Method of Statement throughout the whole work: COMBINATION OF STATEMENTS; COMPLEX FRACTIONS; the general and easy method of disposing of Fractions, etc., etc.

It has been the constant aim of the author to combine UTILITY, BREVITY, and SIMPLICITY; to use such language in all explanations, as could be easily understood; and to present to the public a real improvement.

All such persons as wish to learn a short, certain, and easy method of making general business calculations; without the assistance of a Teacher, will find in this work everything that is calculated to assist them in this desirable undertaking. ASSISTANCE TO THE PRIVATE STUDENT is a peculiar feature of this treatise.

Several hundred COIN PLATES are added, of whose utility it is unnecessary to say anything to those concerned.

*"It is, without doubt, a very useful book." Cincinnati Daily Commercial.*

## RECOMMENDATIONS.

### *From the Wayne County Whig.*

"We again call the attention of our readers to the appointment of Prof. Rainey, to lecture at the Methodist Church, on this Wednesday evening. We had the pleasure of attending the lectures delivered in our place by this gentleman last week. Under ordinary circumstances, the subject of Mathematics is exceedingly dry and uninteresting. To the lectures of Mr. Rainey, we listened with most intense interest, as did all who heard him. He gives to the subject a novelty and interest we have never before witnessed. His new mode of calculation is the shortest, most rapid, and simple of any mode now in use. We entertain the very highest opinion of the practical utility of his plan, and commend it to the favorable notice of students and business men. The following communication was sent us for publication. We endorse readily and fully the commendations it contains. It comes from men well known as worthy of every confidence, and well qualified to judge of what they speak.

"Mr. Editor :—Prof. Rainey, who has been lecturing in our place some days, on his short and beautiful system of practical calculations, intends visiting your town. We take pleasure in commending this matter to your attention, because we feel convinced that he will greatly benefit all who go to hear him. In his lectures in this place, he has proved his system of calculations to be the shortest and the most simple, as well as the most expeditious that we have ever seen. His operations at the blackboard convince every thinking man of brevity and precision, certainty, simplicity, and final satisfaction; while they excite equal wonder and pleasure for their philosophical beauty and grandeur. He throws away, instead of using figures, and works all fractional questions with ease and simplicity.

"Nothing that we have ever seen can equal his calculations in Simple Interest. Instead of a dozen different rules for different rates per cent., he has one simple rule, short and easily understood, for working questions of any conceivable principal, time, and rate per cent. This every practical man knows to be a great convenience. Prof. Rainey has with him a large work in which his system is thoroughly developed; treating of every variety of calculation that can possibly occur among business men, either in Commercial and Mechanical Arithmetic, or in Mensuration. This is an excellent work, and the first we have seen that thoroughly explained these operations. He gives his reasons for a thing first; then deduces the rule. Hence, his rules appeal to common sense. His work in these rules, as well as others, cannot fail to excite admiration and attention wherever intelligent men will take time to investigate. As he has but a short time to stay in your place, we think it well to call the attention of your citizens to the subject, that they may secure the opportunity, and not regret when too late. Let all your people hear him, and not fail to secure his books.

[Signed] JAMES M. POE, }  
CHARLES FISKE, } Teachers, Richmond, Ia.  
WILLIAM AUSTIN, }  
DR. JOHN FRICHT, } Centreville, Ia."  
REV. E. MCCORD, }

\* \* \* \* \* "RESOLVED, 11. That Prof. Rainey's book is just such a work as the learner needs; because every principle and operation is so thoroughly explained and illustrated, that by its investigation, the ordinary reader will be enabled to comprehend and practice the system.

"RESOLVED, 12. That his system of Interest, as taught in his work, is of itself, worth more than his charge for both instruction and his books, and being short and easily understood, is pre-eminently adapted to the counting-house.

[Signed by] P. Y. Wilson, Finley Bigger, John W. Barber, Dr. W. Frame, J. A. Kendall, Dr. A. Norris, and others of Rushville, Ia; by John C. Osborn, Thomas Kirby, G. B. Holland, Dr. Andrews, Esquire Swarr, George B. Norris, and others, of Muncie, Ia; and by Wm. F. Kelso, James McMeans, and twenty others, of Newcastle, Ia."

### *From the Cincinnati Evening Dispatch;*

also,

### *From the Cincinnati Daily Enquirer.*

"RAINEY'S IMPROVED ARABUS. . . . It purports to be 'An explanatory Treatise on the Theory and Practice of Arithmetic and Mensuration,' and from a hasty examination it appears to be of great practical use in making calculations in a great variety of business relations. It is recommended in high terms by teachers and others competent to judge, who have examined it."

# RAINEY'S IMPROVED ABACUS:

AN EXPLANATORY TREATISE ON THE

THEORY AND PRACTICE

OF

ARITHMETIC AND MENSURATION:

IN WHICH THE GENERAL PRINCIPLES INVOLVED IN PRACTICAL CALCULATIONS ARE THOROUGHLY ELUCIDATED AND ILLUSTRATED BY NUMEROUS ANALYTIC AND ABBREVIATED EXAMPLES.

---

BY THOMAS RAINEY.

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ENERGY IS THE PRICE OF SUCCESS.

CINCINNATI:  
E. D. TRUMAN, PUBLISHER.

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1849

## NOTE TO TEACHERS.

The Teacher will observe that this work is devoted exclusively to the development and illustration of the *principles* of numbers, with the introduction of such a number, and variety of examples, only, as subserve this purpose. It is deemed the privilege of the teacher, to present such examples for practice and test, as may best accord with his judgment, and in the highest degree develop the capacities of the learner.

This arrangement presupposes the plan of instructing classes at the *blackboard*, by familiar illustrations, and the occasional test of each pupil's progress, in the presence of the whole class. *A blackboard is indispensable to every good school.*

In Mensuration, every figure explained in the text should be carefully drawn on the blackboard; that the twofold purpose of *illustration* and *drawing* might be subserved at the same time.

It has been deemed useless to treat of the Elements of Arithmetic, as this department of numbers is generally studied in a separate book; and as a considerable number of good elementary works now claim the patronage of the public.

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Entered according to Act of Congress, in the year 1849,  
By THOMAS RAINEY,  
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Book 100

## PREFACE.

It would be both absurd and arrogant for an author, at this day, to present a new text-book on Arithmetic, according to the old and long cherished standards. A large number of accomplished mathematicians and experienced teachers, as well in our own country as in Europe, have plied their talents and energies to this subject with peculiar ability and success; presenting works, perfect in their order and arrangements, and sufficiently intelligible and satisfactory, so far as the old systems are concerned, for all practical purposes.

Although Arithmetic has, until recently, been neglected by scientific men, yet the onward progress of latter-day improvements has necessitated a corresponding advance in this science; until a place is now conceded it among the rational and explicable sciences, instead of among the mere handicraft arts, as hitherto. This advance in the *principles* of the science, has involved new issues. It is found that the ordinary system of statements is too *mechanical, circumstantial, and uncertain*; that the *method* of statement, within itself, precludes that *rational analysis, and thorough demonstration of principles* necessary to the proper appreciation and use of any science; and that a sum of labor is performed, in the practical reduction of calculations, which is not only unnecessary, but which diverts the mind from the proper issues involved in the problem, and leads it into a labyrinth of doubts and obscurities. It is to the remedy of these palpable defects, that our labors are addressed; and to the presentation of a *rational, satisfactory, simple, unique and brief* system of calculations, such as demanded by the practice of ordinary business transactions.

It is likewise designed to furnish the *private student* with an *easy and certain* guide to proficiency in numbers, on such

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*principles*, and in such accordance with *common sense*, as will appear to him reasonable and convincing. Hence, while the *theory* of each department of numbers treated, is clearly elucidated, the *practice* is illustrated and demonstrated by familiar examples, occurring in business; and, too, by a method so short and general in its application, as to admit of demonstration more conclusive and pointed, than when encumbered by the masses of unnecessary multiplication, division, etc., which always attend the solution by the ordinary method.

Throughout the whole treatise, the reader will observe that much importance is attached to the comprehension and proper application of the principles of Proportion; as constituting, chiefly, the basis of all those operations which follow the elementary rules. In accordance with this view, all of the statements in this work, except those of addition and subtraction, are made by Simple, Compound, or Concatenated Proportion; hence, they are unique, and may be easily remembered.

The beautiful theory of Cause and Effect is thoroughly discussed, and applied to Compound Proportion. The theory of Inverse Proportion will, as dependent on Cause and Effect, be found quite different from any hitherto presented.

Much attention has been given to Mensuration, because of its practical utility, and the constant necessity of the application of its principles in active life.

Contrary to the custom of many authors, we have excluded from this treatise all of those sub-divisions of numbers whose explanation depends on algebraic principles; such as the Positions, Alligation, the Progressions, Permutation, Cube Root, etc.; none of which offer any reward for the arduous labor lost in the impossible task of their attainment in Arithmetic. *If one-half the time devoted to these principles, in their arbitrary form in arithmetic, were given to the study of Algebra, the pupil would not only learn a great portion of that beautiful science, but would thus secure the only key to the principles involved in these rules.*

It has been a prime object, *first*, to discuss principles, and

*then deduce practical directions; for rules, without reasons, are ridiculous, and insulting to the inquiring mind.*

No secondary principle has been used in the elucidation or illustration of one that is primary; nor has any principle been anticipated; but each, used in its natural sequence, has been made the basis of a yet higher principle, in such manner, as to *cultivate the reasoning powers of the learner, without embarrassing them.*

Technical phraseology has been avoided, as far as consistent with the requirements of such a treatise; as likewise, *puzzles*, and all giddy theorizing on trivial and unimportant topics, which should be beneath the dignity of a scientific man, although well calculated to please the fancies of a vacant mind: for he who would be a useful man, must be a practical man; and the less acquainted with fascinating chimeras, the better he is adapted to his great purpose.

It is not claimed for this system that *cancellation* can be availed in every solution; but that a great majority of practical questions can be much abbreviated by it; the excellency claimed for the system, is, that while it abbreviates the work, the *statement is so simple, so philosophical*, and the *result, so inevitable*, that no intelligent individual can fail practicing its principles, whether the arbitrary rules be remembered or not.

We shall endeavor to deal mildly with those who, being bound to the old system, as their *hobby*, cannot, or will not, open their eyes to the evident advance of modern improvements; and, therefore, submit our labors to the investigation of those candid and intelligent minds, which are not shackled down to such usages of the past, as are endeared more by habit, than by any rational merit.

T. RAINEY.

-Cincinnati, July, 1849.

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# CHAPIN

RAINEY'S.

## IMPROVED ABACUS.

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### CANCELATION.

ALL arithmetical computations are effected by *increase* and *decrease*, which depend in their relations, on the converse operations of Multiplication and Division. The latter are but abbreviated methods of adding and subtracting. Increase and decrease are the results of the relative difference between different numbers and quantities of the same thing: hence their result depends, in all reckoning, on the great principles of *Ratio* and *Proportion*. Therefore, by Proportion, as the *rationale* of statement, and Multiplication and Division as the mechanical media of reducing such statements to their results, we have in a few words, an epitome of all arithmetic.

As by this, Multiplication and Division are presented as the leading operations of reckoning, we may profitably spend some little time, in ascertaining a *more expeditious* method of determining products and quotients, than by the old, tedious, and circumlocutory formulæ of the books.

When 7 is *multiplied* by 3, the product is 21: this product *divided* by another 3, gives 7 again;

the *is* not changed by the Multiplication and Division: it may, therefore, be inferred, that, *when any number is both multiplied and divided by any other number, the former remains unchanged.* Hence, such multipliers and divisors may be dropped, as useless, and the numbers *canceled*. If the 7 be multiplied by 5, and the product divided by 10, the result will be  $3\frac{1}{2}$ , or  $\frac{1}{2}$  of the 7; because the 5 has only one half the capacity in elevating, that the 10 has in depressing: consequently, the 7 is affected twice as much by Division as by Multiplication. We may therefore divide 10 by 5, and place the quotient 2, on the side of the 10, which shows the relation between these numbers; the one, increasing by multiplication, on the right; the other, decreasing by division, on the left. Again:

Two numbers, as 12 and 16, sustain to each other a relation, that may be expressed by smaller numbers. They may be reduced to such smaller numbers, by extracting a *factor* or figure which has been instrumental in producing the numbers in each case. Thus, 12 is composed of 4 times 3, while 16 is composed of 4 times 4. Here the same factor which has been used in making each number, the 4, may be extracted in each case, leaving 3 in the 12, and 4 in the 16: which shows that  $\frac{1}{3}$  and  $\frac{1}{4}$  are equal to  $\frac{4}{3}$ ; or, that  $\frac{1}{3}$  and  $\frac{1}{4}$  make  $\frac{7}{12}$ . In this case, as in all other cases of factors, the number supposed, must not be written down; and must be contained without a remainder, in some number on both the right and left of the line. Such numbers on the two sides of the line, as

are divided by the supposed factor, may be canceled, and the other constituent factor *must* be set on the side of the canceled number, from which taken.\*

When there are numbers on the right and left, terminated by ciphers, these ciphers may be stricken off in equal numbers, as so many factors of 10. In  $\frac{2}{3} \frac{0}{0}$  we cancel the two ciphers, and leave  $\frac{2}{3}$ ; which is equivalent to extracting 10 in each case. No cipher can be canceled which has a *significant* digit on its right; for its value is qualified by such digit. Numbers on the left of ciphers may be canceled with other numbers on the opposite side of the line; as such numbers are but factors co-operating with the ciphers on the right, to constitute their sum. Ciphers thus isolated indicate 10. From these considerations we conclude, that, to cancel numbers, after they have been arranged on the two sides of the vertical line,

1st. *Cancel all equal numbers on the two sides of the line:*

2d. *Cancel ciphers in equal numbers:*

3d. *Divide, leaving no remainder, from one side of the line into the other, and, vice versa, placing the quotient on the side of the larger:*

4th. *Extract all possible factors from any two numbers occupying different sides of the line, and leave the other constituents of such numbers, on the side of each.*

A few examples in the multiplication of fractions are given, merely to illustrate the

\* Cancel, is from the French *canceller*, which signifies literally, to cross a writing.

application of the foregoing directions. Multiply  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{16}$  of 2, by  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{16}$  of 5. Here, as in the multiplication of all fractions, we place all the Numerators on the right, and all the Denominators on the left. We have not sufficient space to give the theory of these statements, in this little treatise, which, it is designed to devote more particularly to operations in practical business; leaving all the work preparatory to this, to lectures at the black-board, or to the elementary works of others.\*

The following remarks may be proper just here :

The upper part of a fraction is called the *Numerator*; and the lower part, the *Denominator*. The Denominator, from *de*, concerning, and *nomen*, a name, shows the name of the fraction, or the number of parts into which the unit or whole thing, is divided. The Numerator, from *numerus*, number, shows the number of parts, of the size indicated by the Denominator, taken. A whole number is considered a numerator, whose denominator would be 1. *Mixed numbers*, such as  $4\frac{1}{2}$ ,  $3\frac{1}{3}$ , &c., before placed on the line, must be reduced to improper fractions. This is done by multiplying the whole number by the denominator of the appended fraction, and adding in the numerator. The denominator of the number is again used, as the denominator of the improper fraction. Thus, in  $4\frac{1}{2}$ , twice 4 make 8, and 1 makes  $\frac{1}{2}$ . In  $3\frac{1}{3}$ , five times 3 are 15, and 1 is  $\frac{1}{3}$ . Hence,

\* Day and Thomson's Practical and Higher Arithmetics.

9 and 16 are the numerators, although they are larger respectively, than their denominators.

Fours equal : 5 into 10, twice ; this 2 equals 2 opposite : 8 into 24, 3 times, while 3 times 3 on the right make 9, which goes into 18 on the left twice : now twice 6 on the left equals 12 on the right : 5 into 35 seven times, and 7 into 14 twice ; this 2 into 16 eight times, on the left : we have remaining on the left 8, and on the right 5 ; making  $\frac{8}{5}$ , Ans.

$$\begin{array}{r|l}
 \begin{array}{r}
 4 \quad 3 \\
 8 \quad 5 \\
 2-10 \\
 6 \quad 5 \\
 8-16 \\
 7-35 \\
 2-14 \\
 2-18
 \end{array}
 &
 \begin{array}{r}
 3 \\
 5 \\
 9 \\
 5 \\
 12 \\
 12 \\
 14 \\
 24 \\
 5
 \end{array}
 \\
 \hline
 85 & \\
 \hline
 8 \text{ Ans.} &
 \end{array}$$

After canceling as far as practicable on the two sides of the line, multiply continuously together all the terms on the right, for a new numerator, and all the terms on the left, for a new denominator. If nothing remains on the right, *one is understood*. If the number on the right be smaller than the number on the left, the answer is a fraction ; of which, in all cases, the right is the numerator ; but, if the number on the left be smaller, the right must be divided by the left : in such case, the answer will be a mixed number.

What will  $7\frac{1}{2}$  lbs. iron come to at  $2\frac{2}{3}$  cts. per lb.? Here  $7\frac{1}{2}$  make  $\frac{15}{2}$ , and  $2\frac{2}{3}$  make  $\frac{8}{3}$ .

We place the numerators on the right, and the denominators on the left. Five into 15, three times ; and 2 into 12 six times ; while six times 3, the only remaining numbers, make 18 cts., the answer.

$$\begin{array}{r|l}
 2-15-3 & \\
 5-12-6 & \\
 \hline
 18 \text{ cts.} &
 \end{array}$$

Multiply,  $\frac{12}{5}$  of a yard of cloth, by  $\frac{1}{4}$  of a dollar per yard, thus,

$$\begin{array}{r|l} 5-25 & 12-3 \\ 4-10 & 12-3 \\ \hline 20 & 9 \\ & 20 \end{array}$$

Here, we must suppose some factor, before the numbers can be reduced. Let us take 5, which goes into 25 five times, and into 15 three times; again: 4 into 16 four times, and into 12 three times; we have 5 and 4 as factors, neither of which has been placed down; only their quotients. Now 3 times 3 on the right, and 4 times 5 on the left, make  $\frac{2}{5}$  of a dollar.

What will  $2\frac{1}{2}$  yards of gambroon come to at 1.20 cents per yard?

$$\begin{array}{r|l} \$21 & \\ 120-15 & \\ \hline \$3,15 & \end{array}$$

Eight is contained in 120, 15 times; and this number multiplied by 21 on the same side, gives \$3,15 cents for the answer.

If  $\frac{3}{5}$  of a farm are divided among 4 heirs, how much will each get?

In this instance the dividend,  $\frac{3}{5}$ , must be placed on the right of the line, and 4, the divisor, on the left; thus,

$$\begin{array}{r|l} 15 & 4-2 \\ 4 & \\ \hline 15 & 2 \\ & 15 \end{array}$$

A fractional number occupies the right or left of the line, when its *numerator* is on the right or left. Let the numerator be located first; then, the denominator is merely placed opposite. If I direct the pupil to place  $\frac{9}{17}$  on the left of the line, he must place the 9 on the left only, and the 17 on the right. The numerator always indicates the locality and value of the fraction.

Divide  $\frac{2}{3}$  of  $\frac{3}{4}$  of 20, by  $\frac{7}{10}$  of  $\frac{5}{6}$  of  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $4\frac{1}{2}$  of  $\frac{4}{5}$ .

The numerators of the dividend are placed on the right, and those of the divisor on the left, with all the denominators opposite their respective numerators.

Four into 20 five times, and 5 equals 5 on the left; 9 into 18 twice, and twice 2 on the right make 4, which goes into 12 on the left 3 times; 5 into 25 five times, and this 5 again into 10 on the right twice; 8 into 40 five times; 3 into 9 three times; 3 equals 3; 2 and 8 remain on the right, and 5, 3, and 7 on the left, which multiplied separately give  $1\frac{1}{2}$ .

$$\begin{array}{r|l}
 4\cancel{2} & \\
 8 & \\
 \hline
 7\cancel{1}\cancel{0}-2 & \\
 5 & \\
 3-\cancel{1}\cancel{2} & 5 \\
 5-\cancel{2}\cancel{5} & \cancel{1}\cancel{8}-\cancel{2} \\
 5-\cancel{3}\cancel{5} & 2 \\
 5-\cancel{4}\cancel{0} & 5 \\
 \hline
 105 & 16 \\
 \hline
 & 105
 \end{array}$$

From the foregoing we may deduce the following directions:

*To multiply fractions; place all of the numerators, both of the multiplicand and multiplier, on the right of the vertical line, and all the denominators on the left.*

*To divide fractions; place the numerators of the dividend on the right, and those of the divisor on the left, with the respective denominators of each opposite their numerators.*

# COMPLEX FRACTIONS.

(It is not designed in this short treatise to devote much space to either the theory or practice of fractions; as it is believed that there are very many elementary works accessible, which do entire justice to this department of numbers. We shall merely introduce a page or two on complex fractions, for the consideration of teachers, that we may present a short and simple method of using them, which is not found in the books.)

Division of ordinary fractions leads to the consideration of those that are *complex*. The

doctrine may be advanced, that *to increase the terms of a fraction, so as not to change its value, multiply both the numerator and denominator by the same number.*

If  $\frac{1}{2}$  be multiplied by 2, it is made  $\frac{2}{4}$ . Now, if instead of  $\frac{1}{2}$ , the fraction were  $\frac{3\frac{1}{2}}{4}$ , we might double the numerator  $3\frac{1}{2}$ , by multiplying by the special denominator 2, and adding in the special numerator 1; thus making  $\frac{7}{2}$  of the numerator.

But if this numerator is thus increased by the 2, the denominator should be likewise: hence the 4 is multiplied by the same 2, making 8, by which we have  $\frac{7}{8}$ , which is equivalent to  $\frac{3\frac{1}{2}}{4}$ . Both terms of the fraction are increased by the same number, the denominator of the fraction annexed to the numerator. After reducing the numerator, it may be expressed thus:  $\frac{7}{2 \times 4}$ , showing that the two denominators have been thrown together, and may, consequently, be combined in multiplication.

It appears thus, that if we would reduce complex to simple fractions, and at the same time to the lowest term, we should *multiply each term of the fraction by the denominator or denominators of the fraction or fractions annexed to the numerator or denominator, or both, adding in at each separate multiplication, the given numerator or numerators.* For example, in the frac-

tion  $\frac{3\frac{1}{2}}{2\frac{1}{2}}$ , we multiply first by the denominator of the numerator, 2. Twice 3 are 6, and one, the numerator, added, makes 7, or  $\frac{7}{2}$ : now we multiply the denominator  $2\frac{1}{2}$ , by the same 2;—twice 2 are 4, and twice  $\frac{1}{2}$  are 1, making  $4\frac{1}{2}$ . This  $4\frac{1}{2}$  must be placed under the 7, thus,  $\frac{7}{4\frac{1}{2}}$ . We have now reduced the

complex fraction in the numerator by multiplying it and the whole denominator below, by the denominator 2 above, and proceed to that of the denominator. Taking the new fraction  $\frac{7}{4\frac{1}{2}}$  to work on, we say 3 times 4 are 12 and 2 are 14, for a new denominator, and 3 times 7 are 21, for a new numerator; thus,  $\frac{21}{14}$ . This reduced to its lowest term, by dividing the numerator by the denominator, gives  $1\frac{1}{2}$  for the answer.

It is seen, therefore, that in each case, it is necessary to reduce both the numerator and the denominator to a mixed number. After this is done, knowing that the denominator of a fraction is the divisor, we may deduce the following rule:

*To reduce complex to simple fractions, of the lowest term: Reduce both terms to an improper fraction, and place the numerator of the numerator on the right, and the numerator of the denominator on the left, with their respective denominators opposite.*

Let us reduce in this way  $\frac{3\frac{1}{2}}{2\frac{1}{2}}$ . The numerator  $3\frac{1}{2}$  is first reduced to  $\frac{7}{2}$  and placed,

the numerator on the right of the line, and the denominator on the left; thus,

Next reduce the denominator  $2\frac{1}{2}$  to an improper fraction, and divide by it, placing the 7, or numerator, as in other cases of division, on the left. Seven equals 7; and 2 is contained in 3 one and a half times, which is the answer, as above.

By this process, the complex is not only reduced to a simple fraction, but to the lowest term of that fraction; all of the factors being excluded in the canceling: while it is plain and intelligible, and at once proves itself necessarily correct to all who take the trouble to know that the numerator of a fraction is the dividend, and the denominator the divisor.

In the same manner that other fractions are multiplied, we multiply these, by placing on the right the numerators, both in the multiplicand and multiplier. Hence,

*To multiply complex fractions and reduce them to their lowest term: Place the numerators of the numerators, both of the multiplicand and multiplier, on the right; the numerators of the denominators on the left; and all respective denominators opposite.*

Multiply  $\frac{8\frac{1}{2}}{4}$  by  $\frac{4\frac{1}{2}}{1\frac{1}{2}}$ . In the first place  $8\frac{1}{2}$  make  $2\frac{1}{2}$ , which are placed on the right, while 4, the denominator, is placed with the 8 on the left. Under this are placed the  $4\frac{1}{2}$ , or  $2\frac{1}{2}$ , the other numerator; and  $\frac{2}{3}$ , the denominator, is placed on the left; thus,

Four times 6 on the left equal 24 on the right. The answer is  $8\frac{1}{2}$ .

$$\begin{array}{r} 3 \overline{) 25} \\ \underline{24} \phantom{0} \\ 1 \phantom{0} \\ \underline{18} \phantom{0} \\ 6 \phantom{0} \\ \underline{6} \phantom{0} \\ 0 \end{array}$$

Again: Multiply  $\frac{7}{2}$  by  $\frac{4}{3}$ . Thus,  $\frac{7}{2}$  is divided by  $\frac{4}{3}$ , and this is multiplied by  $\frac{3}{4}$  and divided by  $\frac{4}{3}$ .

Three is contained in 6 twice, and twice 4 equals 8 on the left. Seven equals 7, while 5 is contained in 9  $1\frac{1}{2}$  times.

$$\begin{array}{r} 3 \overline{) 7} \\ \underline{6} \phantom{0} \\ 1 \phantom{0} \\ \underline{3} \phantom{0} \\ 4 \phantom{0} \\ \underline{3} \phantom{0} \\ 1 \phantom{0} \\ \underline{1} \phantom{0} \\ 0 \end{array}$$

Division of complex fractions may be performed as in the division of other fractions, by inverting or placing on the left of the line the numerator of the divisor. Hence,

*To divide one complex fraction by another: Place the dividend on the right and the divisor on the left.*

Divide  $\frac{4\frac{1}{2}}{7\frac{1}{2}}$  by  $\frac{2\frac{1}{2}}{3\frac{1}{2}}$ . Here we place  $4\frac{1}{2}$  on the right, and  $7\frac{1}{2}$  on the left; and dividing by  $2\frac{1}{2}$ , place it on the left, while its denominator  $3\frac{1}{2}$  is placed opposite.

It appears that all of the numbers equal: hence, *one* is understood on the right for the quotient.

$$\begin{array}{r} 3 \overline{) 4} \\ \underline{3} \phantom{0} \\ 1 \phantom{0} \\ \underline{1} \phantom{0} \\ 0 \end{array}$$

It is foreign to our purpose to fill this little volume with examples for the reader to solve; hence, after pointing out clearly the principles

governing *statements* and *solutions*, we shall give him the privilege of working such questions as his experience and business may suggest; a few examples, however, will be introduced, to familiarize him with canceling and using figures by this system.

Multiply  $3\frac{1}{2}$  lbs. cheese, by  $8\frac{1}{2}$  cents per lb.

Multiply  $\frac{1}{2}$  of  $\frac{1}{10}$  of 20, by  $\frac{2}{3}$  of  $\frac{1}{4}$  of  $\frac{1}{10}$  of  $\frac{1}{2}$  of 10 of 180.

Multiply  $\frac{3\frac{1}{2}}{\frac{1}{2}}$  of 40 by  $\frac{24}{4\frac{1}{2}}$ , and divide by  $\frac{1}{2}$  of  $\frac{1}{10}$ .

Divide  $\frac{1}{2}$  by  $\frac{1}{10}$ , and multiply it by  $\frac{1}{2}$  of 20.

The examples above are deemed sufficient to enable the learner to proceed with ease, after the questions are stated, without a teacher. If the numbers on the two sides of the line are such as cannot be canceled, they must be multiplied and divided, and the result will be the same. The vertical line will be explained in Simple Proportion.

## SIMPLE INTEREST.\*

The following method of casting interest is designed for every rate percent. Many of the questions wrought under this head, can be wrought with less than half the number of figures here required, by the short 6 per cent., and other methods which will follow.

SIMPLE INTEREST is an allowance made for the use of money, and is different in different countries. It is supposed that, as in one year the products of the soil, on which all other profits are primarily based, yield their regular

\* I am indebted to the Rev. William McGookin, of Ohio, for many important suggestions on simple interest.

increase, so it is reasonable that a borrower of money should be required to refund the amount borrowed, annually. Hence, the unit of time for which money is lent, is one year; called *per annum* or *by the year*, from the Latin words, *per*—by, and *annum*—a year. The charge of a specific price or bonus for the use of money, requires that a sum be established, as a *general sum*, on which, in one year, the specified, legal interest shall be charged; so that a larger sum would receive proportionally more, and a smaller sum proportionally less interest; while likewise a greater or less length of time than one year, would make the interest proportionally more or less. This specified sum is 100; a number chosen because easily used in dividing; being the round product of *two decimates*. The 100 is called *per centum*, from *per*—by, and *centum*, a hundred. Hence *per centum per annum* means, by the hundred, by the year. *Cent.* is a contraction of *centum*.

Most of the States of our Union have established 6 per cent. *per annum*, as the legal rate of interest. Now, it is manifest that if *per centum* or 100, in *per annum* or 1 year, that is, if 100 dollars in one year, gain 6 dollars interest, a larger or smaller number of dollars will gain more or less than 6 dollars, in one year; while likewise, \$100 in a greater or less time than one year, would gain more or less than 6 dollars interest. If \$100 in one year gain 6 interest, 200 would gain twice 6, or 12; and if 200 in one year gain 12, in two years it would gain twice twelve; or in one half of a year the half of 12, or 6. So we perceive that the

interest on a sum of money for a given period of time depends on the relation that such sum bears to 100, as well as the relation of the time to 1 year. These relations are properly ascertained by Compound Proportion; but we will here present them in the form of two connected simple proportions. What is the interest on \$50 for six months, at 6 per cent.? that is, if \$100 in 12 months gain \$6 interest, what interest will \$50 gain in 6 months? Now we place the demand, \$50 and 6 months, on the right of a line, and the terms of the *same name*, 100 dollars and 12 months, as the supposition, opposite these, on the left. When we place dollars opposite dollars thus, it is to get the ratio between the two numbers of dollars: the same is the case with the months, which are placed opposite. The ratio between 100 and 50 is  $\frac{1}{2}$ ; and between 12 months and 6 months  $\frac{1}{2}$ ; so that the two ratios multiplied together, that is  $\frac{1}{2}$  times  $\frac{1}{2}$ , make  $\frac{1}{4}$ ; hence, the \$6 interest multiplied by this  $\frac{1}{4}$ , is reduced to  $1\frac{1}{2}$ , which is the result; thus,

$$\begin{array}{r|l}
 2-100 & 60 \\
 2-12 & 6 \\
 \hline
 & 6-3 \\
 & \$1\frac{1}{2}
 \end{array}$$

The ratio of these numbers compared, is obtained as by a *fulcrum* and *scale*. Hence, 50 is contained in 100 twice, and 6 in 12 twice, and this 2 in 6 three times, and 2 on the left in 3 on the right  $1\frac{1}{2}$  times, which is  $1\frac{1}{2}$  dollars, the answer. Suppose the time above, instead of being 6 months, were 6 days; then we could not place one year opposite it in the form of 12 months; but only in the form of 360 days: for, to obtain the ratio of time that 6 days bear to a year, we

must compare days with days, not with months: hence if the time were 6 days, we would place 360 days, which make a year, opposite; or which is the same thing, 30 days opposite, to make a month, and 12 months to make a year: these two numbers make 360, the number of days in an interest year. Let us find the interest for 6 days; thus,

We must place on the left opposite the specified time, 1 year, 12 months, or 360 days, just as the time *specified*, may be in years, months or days. Here again, 50 into 100 twice: 6 times 6 on the right equal 36 on the left; leaving the cipher or 10 to be multiplied into the 2, making the denominator 20. One being understood on the right, the answer is  $\frac{1}{20}$  of a dollar; equal to 5 cents.

$$\begin{array}{r|l} 2-100 & \$ \\ 360 & \$ \\ \hline 20 & 1 \\ & 20 \end{array}$$

We see then, that 100 and the time, either in years, months or days, go to the left; while we place the sum on which the interest is to be obtained, the time, and the *rate*, all on the right.

Now if the rate be 6, or 10, or any other per cent., it is so many hundredths; 100 being the denominator, while 6 or 10, &c., is the numerator. This denominator is composed of two decimals. Decimal is from the Latin word *decem*, which means ten; hence, the decimal is the tenth part of a unit. Two of these decimals, or the tenth times one tenth, make one one-hundredth: hence, any two decimal factors, express so many hundredths. It is necessary in expressing a decimal fraction, that there

be one figure less in the numerator than in the denominator: and the denominator 100, containing 3 figures, we always consider that there are but two in the numerator or rate. Then, if we consider the rate per cent. two decimals, every thing multiplied into it must be made 100 times smaller: that is, if dollars be multiplied by the rate, they become hundredths of dollars, or cents: and if cents be thus multiplied by the rate, or two decimals, they become hundredths of cents. Now, in the first question wrought, let us make this rate two decimal factors, by dropping the 100 at the left. It is unnecessary here to place a cipher at the left of the 6, to show that the rate is composed of two decimals: this will be understood.

$\begin{array}{r} 50 \\ 2-12\cancel{6} \\ \hline \end{array}$	$\begin{array}{r} 50 \\ \cancel{6}3 \\ \hline \end{array}$
\$1,50	

Certainly, when we multiply the \$50 by this rate per cent., 6, it is made 100 times smaller than dollars, and becomes cents; so that the answer is 150 cents. We

will therefore cut off two figures at the right of the result for cents. Suppose again, we consider this 50 cents, instead of 50 dollars: then the cents being multiplied by the rate, become hundredths of cents; so that in the answer, we cut off two for hundredths of cents, two more, if we have them, for cents, while the remaining figures at the left, are dollars. But having only 3 figures, the answer is 1 cent and 50 hundredths; or  $1\frac{1}{2}$  cents. Let us get the interest on \$60, for 317 days, at 6 per cent. We call the \$60 here, as we call the sum in all other cases, the PRINCIPAL—the 317 days, the TIME, and the 6 per cent. per annum, the

**RATE.** We place these numbers on the right, thus,

and as the time is in days, we place 30 and 12 opposite, or 360, which is the same thing; and again dispense with the 100. If we dispense with this 100 on the left, the answer will be 100 times smaller than the principal; and this being dollars, the answer will be cents. Cyphers equal: 6 into 12 twice: and twice 3 on the left, equal 6 on the right. We have 317 left, and conclude, that the answer is 3 dollars and 17 cents. If the principal were 60 cents, the answer would be 3 cents and 17 hundredths. From this we conclude, that, *When the Principal is dollars, the answer is cents; and when the principal is cents, the answer is hundredths of cents.*

What is the interest on \$80, for 9 months, at 7 per cent.? Here, we place the P. T. & R. as before, on the right; and place 12 only, on the left; because the time is in months, or  $\frac{9}{12}$  of a year.

We use the factor 4, which goes into 12 three times, and into 8 twice: 3 into 9 three times: now  $3 \times 2 \times 7 \times 10$  make 420, or \$4 and 20 cents. Were this time 9 days, we would place 30 with 12 on the left. Were the principal 80 cents, the answer would be four cents and  $\frac{20}{100}$ .

What is the interest on  $37\frac{1}{2}$  cents for 18 days at  $7\frac{1}{2}$  per cent.? The principal and rate being mixed numbers, must be reduced to improper fractions; and the numerators placed

$$\begin{array}{r|l} & 30 \\ 30 & 317 \\ \hline & 317 \\ & \$3,17 \end{array}$$

$$\begin{array}{r|l} & 80 \\ 12 & 3 \\ \hline & 7 \\ & \$4,20 \end{array}$$

on the right, with their denominators on the left. *The numerator of a fraction in all cases, occupies the same place that otherwise the whole number would: while the denominator is invariably placed opposite.* Hence again, the time being days, we place 30 and 12 on the left. The principal is cents;  $\frac{7}{5}$  cents; hence, the answer will be hundredths; and as such, we will strike off two numbers for hundredths, and two for cents.

$$\begin{array}{r}
 2,75 \\
 \cancel{2} \cancel{0} \cancel{1} \cancel{5} \cancel{3} \\
 4 \cancel{1} \cancel{2} \\
 \hline
 2 \cancel{1} \cancel{5} \\
 \hline
 16 \overline{) 225} \\
 \hline
 14 \frac{1}{16}
 \end{array}$$

Fifteen into 30, twice, and 2 into 18, 9 times: The factor 3 into 9 three, and into 12, four times;  $4 \times 2 \times 2$  are 16 on the left, and  $3 \times 75$  on the right, are 225; which divided by 16 gives  $14 \frac{1}{16}$ . The

answer is no dollars, no cents, and  $14 \frac{1}{16}$  hundredths cents. Such examples as this are scarcely of any practical value, and only show the full extent of the theory and practice of interest by this system.

What is the interest on \$600,60 cents, for  $3\frac{1}{2}$  years, at  $4\frac{1}{2}$  per cent.? We make the  $3\frac{1}{2}$  years,  $\frac{1}{3}$ , and place the 10 on the right, and 3 on the left, and divide by nothing, except the denominator. All that we divide by the numbers 12, and 12 and 30 for, is to reduce the time to years; hence, when the time is already in years, division by any number becomes unnecessary, except by such denominators, as from mixed numbers, may fall on the left; which is the case with 3 and 2 in this example. Here, the  $4\frac{1}{2}$  make  $\frac{9}{2}$  per cent.

Two into 10 five, and 3 into 9 three times: now  $3 \times 5 \times 600,60$  make 900900. We cut off two for hundredths, and two for cents: hence, the answer \$90,09 cents, and no hundredths. This answer is in hundredths, because the principal is in cents.

$$\begin{array}{r} 600,60 \\ 3 \overline{) 18} - 5 \\ 3 \overline{) 9} - 3 \\ \hline \$90,09,00 \end{array}$$

What is the interest on \$600, for 3 years, 6 months and 20 days, at 6 per cent.? Here it is necessary to reduce all the years to months, and add in the given months; and likewise reduce the days to the fractional part of a month, and add such fraction to the months. In three years there are 36 months, and 6 more added, make 42 months. Now, 20 days are  $\frac{2}{3}$  of a month, which, canceling the two ciphers, makes  $\frac{2}{3}$ . The time, therefore, is  $42\frac{2}{3}$  months, which make  $128\frac{2}{3}$  months. We place this 128 on the right, and 3 on the left. The time now being in months, we divide by 12 only.

Six into 12 twice, and twice 3 on the left, equal 6 on the right: we consequently draw down the 128, and annex the two ciphers, making the answer \$128,00. Suppose the time had been 1 year, 1 month, and 10 days. One year and 1 month make 13 months: 10 days are  $\frac{1}{3}$  or  $\frac{1}{3}$  of a month: consequently the time is  $13\frac{1}{3}$ , or  $13\frac{1}{3}$  months. Here, 40 should go to the right, and 3 to the left, with 12. Again: Suppose the time 6 months and 15 days: These 15 days are  $\frac{1}{2}$  or  $\frac{1}{2}$  month; so that the time is  $6\frac{1}{2}$ , or  $6\frac{1}{2}$  months. Here, again, 12 should be placed on the left. Suppose the time 2 years, 9 months

$$\begin{array}{r} 600 \\ 3 \overline{) 128} \\ 3 \overline{) 12} - 12 \\ \hline \$128,00 \end{array}$$

and 25 days: the 25 days make thus,  $\frac{25}{30}$ , equal to  $\frac{5}{6}$  of a month; and two years and 9 months, make 33 months, which, with the  $\frac{5}{6}$  annexed, is  $33\frac{5}{6}$  or  $33\frac{2}{3}$  months. Suppose the time 27 days: this would be  $\frac{27}{30}$ , or  $\frac{9}{10}$  of a month, to be annexed to all the months. Were it 28 days, it would be  $\frac{28}{30}$ , or  $\frac{14}{15}$  of a month. Nine days would be  $\frac{9}{30}$ , or  $\frac{3}{10}$  of a month: so would 8 days be  $\frac{8}{30}$ , or  $\frac{4}{15}$  of a month. Suppose the time were three months and 29 days. Most business men would call this 30 days: but to be accurate, we would multiply the months by 30 and add in the 29. Thus, the whole time would be reduced to 119 days: and we would consequently divide by 30 and 12. When the days make a number that cannot be reduced to the fraction of a month, to secure entire accuracy, the years must be reduced to months, and all the given months added in; then, these months must be reduced to days, and the given *days* added in.

What is the interest on \$50 for 1 year, 3 months and 5 days, at 6 per cent.? One year and 3 months make 15 months: 5 days are  $\frac{1}{6}$  of a month: hence  $15\frac{1}{6}$ , or  $15\frac{1}{6}$  months, is the time. We divide by 12 only.

$\begin{array}{r} \$ \quad 50 \\ \quad 91 \\ 12 \overline{) 91} \\ \hline 12 \overline{) 4550} \\ \hline \$ 3,791\frac{1}{6} \end{array}$	<p>Sixes equal: we have 12 on the left, and on the right <math>50 \times 91</math> which makes 4550. This divided by 12 gives for answer \$3,791<math>\frac{1}{6}</math> cents.</p>
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From the foregoing principles and operations, we are justified in making the following

## SUMMARY OF DIRECTIONS,

For working Interest of any conceivable Principal, Time, and Rate.

*Place the Principal, Time, and Rate, on the right of the vertical line ; and if the time is days, place 30 and 12 on the left : if the time is months, place 12 only, on the left : and if the time is years, place nothing on the left.*

*If the Principal, Time, or Rate is a mixed number, reduce it to an improper fraction, and place the numerator on the right, with the denominator on the left.*

*When the Principal is dollars, the answer is cents: in such case, two figures must be cut off for cents : when the Principal is cents, the answer is hundredths of cents: here, cut off two figures, commencing at the right, for hundredths, two more for cents, and the remainder at the left is dollars. The figures thus cut off for cents, hundredths, &c., must be whole numbers ; while any existing fraction will be only a fractional part of such cents or hundredths.*

*When the time is months and days, or years, months and days, reduce the years to months, and add in all the given months : then reduce the days to the fractional part of a month, and annex this fraction to the whole number of months: reduce all to an improper fraction, and place the numerator on the right, and the denominator on the left. In such case, divide by 12 only. If the time cannot be reduced to the fractional part of a month, reduce the whole time, years, months and days, to days, and divide by 30 and 12.*

*If the time is years and months, reduce the months to the fractional part of a year: add to the years: reduce all to an improper fraction, and divide by the denominator only.*

If the answer to a question be \$80, 20 cents and  $38\frac{1}{2}\frac{2}{3}$  hundredths, and it were written thus, 80.20.38 $\frac{1}{2}\frac{2}{3}$ , it would be wrong to cut off either the 18 or 29 by itself, or both together, for the denomination of hundredths; for they make only the  $\frac{1}{2}\frac{2}{3}$  part of *one* one hundredth part of a cent. Hence, to strike off cents or hundredths of cents, place the separatrix between integral numbers only.

The use of 360 days to the year, may be by some thought singular; but it grows out of the standard commercial usage, 30 days to the month, and 12 months to the year. The business year, the civilized world over, is called 360 days. If, however, any wish to use the 365, they can easily do so by substituting 365 on the left, for 30 and 12. The difference in the result is only  $\frac{1}{36}$  part. In banks, the interest is always reckoned for three days more than the time specified by the borrower, which are called *days of grace*, or days given the borrower to allow for any accidents or exigencies which may prevent the money being funded at the close of the discounting period. Grace means *gift*. *Really* the three days are *not* days of grace; for interest is reckoned on them as part of the general discounting time. Banks, too, charge more than the legal rate of interest; on what principle of ethics, however, I have never been able to learn. If the rate be 6 per cent., the note for one year, is given for

\$100; and the interest on \$100, which is \$6, is deducted from the money when issued to the borrower; so that he gets only \$94. Now here, he pays \$6, not for the use of 100, which would be equitable, at 6 per cent., but for 94. The interest on \$94, the sum that the borrower receives, at 6 per cent., will not be \$6; so that he loses clear the difference between the \$6, and the sum of interest that 94 would gain. This difference is quite important in heavy transactions.

It is frequently impossible to cancel in questions of interest; when this is the case, all the numbers on the right must be multiplied together for a dividend, and all on the left for a divisor: after which the former must be divided by the latter. Some would ask, "what benefit in working interest in this way, if at times, it is necessary to multiply and divide, as in the old system?" It is because all questions, whatever be the principal, time or rate, can be wrought by this one, simple rule, without a separate rule for every varying per cent.; and which rule itself, is based on a principle so remote, as seldom to be seen by the ordinary arithmetician: because the statement can be easily made and understood by any ordinary mind; because the work is unique and systematic; and because in most cases the work can be greatly abbreviated by canceling. We give but two other examples, and these without the work. What is the interest on \$800, for 2 years, 8 months and 15 days, at 10 per

$$\begin{array}{r}
 800 - 4 \\
 2 \overline{) 65} \\
 3 - 12 \overline{) 10} \\
 \hline
 3 \overline{) 65000} \\
 \hline
 \$216,66\frac{2}{3}
 \end{array}$$

cent.? Here then, the time makes  $\frac{3}{4}$  months.

$\begin{array}{r} 30 \overline{) 10,000} \\ 12 \overline{) 90} \\ 4 \overline{) 1} \\ \hline \$6,25 \end{array}$	<p>What is the interest on \$10,000 for 90 days, at <math>\frac{1}{4}</math> of 1 per cent.? This time may be placed on the line as 90 days 3 months, or <math>\frac{3}{4}</math> of a year. In the first case, we would divide by 30 and 12; in the second by 12 only, and in the last by nothing.</p>
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### INTEREST AT SIX PER CENT.

Interest at 6 per cent. has long since been reckoned by dividing by 60, when the time was days, or when months, multiplying the principal by half their number. This process is very easy, when the number of months is even, and the half can be found without making it necessary to multiply the principal by a mixed number; but when the time is an odd number, or has years, months and days; or when the principal is a mixed number, it is difficult, by the process ordinarily pursued, to use the fractions, and ascertain the precise result. Hence, the time is generally made too great or too little, and many fractions are thrown away; whereas by the use of the vertical line, and the consequent advantage of placing the numerators and denominators on its two sides, this difficulty is entirely obviated; so that if the numbers on the two sides cannot be canceled, they can at least be multiplied and divided, as by the old method. By this method, therefore, a clear gain is made of

all numbers that *can* be canceled, as well as greater ease and perspicuity in the statement.

We find by the first method presented in this work, that the interest on *one dollar*, for *60 days*, at *6 per cent.*, is *one cent.* It is likewise the same for *2 months*, thus :

$$\begin{array}{r} 1 \\ 30 \overline{) 60 - 2} \\ 12 \overline{) 6} \\ \hline 1 \text{ cent.} \end{array}$$

$$\begin{array}{r} 1 \\ 2 - 12 \overline{) 2} \\ \hline 1 \text{ cent.} \end{array}$$

In the former case, the time being *60 days*, 30 and 12 are used on the left; in the latter, the time being *2 months*, 12 only is used. The result is the same.

If *one dollar*, as above, give 1 cent interest in 60 days, it will in 6, which is the tenth part of 60 days, give the tenth part of a cent, or *one mill.* The fact is therefore established, that

*One dollar in 6 days, at 6 per cent., will gain ONE MILL interest; and ONE DOLLAR in 2 months at 6 per cent., will gain ONE CENT interest.* Hence, we are justified in making the following statement in Proportion: If 1 dollar in 6 days on the left, give one mill interest, last on the right, how many mills will any other number of dollars and days give on the right? Or, if one dollar, in 2 months on the left, give one cent interest, last on the right, how many cents will any other number of dollars and months give, on the right?

What is the interest on \$40 for 240 days? Here, the principal and time are placed for a

demand, on the right, 1 dollar and 6 days, for the same name on the left, and 1 mill, last on the right, for the denomination of the answer:

$$\begin{array}{r} 140 \\ \$240-4 \\ 1 \\ \hline 1,60,0 \end{array}$$

Thus. The answer is consequently, mills; hence, *one figure* must be cut off for mills, and all at the left of it are cents and dollars. The answer is one dollar, 60 cents and no mills.

The ones are placed on the two sides of the line, merely to indicate the *proportion* in the statement. They are unnecessary in practice, and may be dropped in the statement and calculation.

$$\begin{array}{r} 20 \\ 6794 \\ 37940 \\ \hline 2,94,6\frac{2}{3} \end{array}$$

What is the interest on 20 cents for 794 days? Two is contained in six three times, which we can divide by, no farther, and consequently bring down on the left. The 794 are multiplied by 10, by merely appending the cipher. In this case, the principal is cents; hence, the answer is 1000 times smaller than if it were dollars, and is consequently thousandths of cents. We strike off one figure for thousandths, two more for hundredths of cents, and the remaining figure is cents; hence the answer, 2 cents, 94 hundredths, and  $6\frac{2}{3}$  thousandths. The business man everywhere, would call this 3 cents. By this it is seen that, *when the principal is dollars, the answer is mills, and when the principal is cents, the answer is thousandths of cents.*

It is quite preferable that the answers should be in dollars, cents, and hundredths of cents. To effect this, when the time is days, and we divide by 6, as in the foregoing, we

may cut off and throw away at the right of the answer, *one figure*, for mills or thousandths of cents, as useless; and the answer will be cents or hundredths, according to the denomination of the principal. It must be remembered, that this figure is thrown away at the right of the answer, only when the time is days.

What is the interest on 680,20 cts. for 93 days?

680,20	— 3401
2-6 31	10203
	\$10,54,31¢ Ans.

Here, the factor 3, is contained in 6 twice, and in 93 thirty-one times; this 2 in 6802 is contained 3401 times. We multiply the latter number by 31, by using the 3 only, not setting it down, but placing its product one move to the left of the unit's place, and adding the two numbers. To this sum we annex the cipher. One figure, the cipher, is thrown off, which leaves the answer 10 dollars, 54 cents, and 31 hundredths. The long method of multiplying by 31 would be quite as easy for most persons as that used above. Or the two original numbers 680,20 and 93 could be multiplied, and their product divided by the 6, producing the same result.

It may be remarked here, that whenever it is necessary to multiply by the numbers 21, 31, 41, 51, 61, 71, 81, 91, the left figure only, may be multiplied by, and its product removed one figure to the *left* of the unit's place, and the two numbers added. Likewise to multiply by 13, 14, 15, 16, 17, 18 and 19, use the right hand figure only, placing the product one move to the right, and add as before. In

neither of the cases do we write the number multiplied by.

What is the interest on  $87\frac{1}{2}$  cents, for 120 days? In this instance we may place the principal on the line as a mixed number  $1\frac{1}{2}$ , as in the annexed example, or we may make the  $\frac{1}{2}$  cents, 5 decimals, and place it down as 87,5, thus,

$$\begin{array}{r} \$175 \\ \$120 \\ \hline 1,750 \end{array}$$

$$\begin{array}{r} 87,5 \\ \$120-2 \\ \hline 1,75,00 \end{array}$$

The result will be the same, except that, in the latter case, one figure must be cut off for the decimals; then the figures remaining may be treated as usually. In the first question, 6 times 2 equal 12 on the right; in the 2d, 6 into 12 twice, and twice 87,5 are 1750, which with the 10 annexed, becomes 17500. The result is the same, after cutting off the decimal in the latter case, and afterwards one figure in each case for the thousandths. Hence, the answer, one cent and 75 hundredths.

What is the interest on  $287,37\frac{1}{2}$  cents, for 80 days? We give the  $\frac{1}{2}$  cent here the decimal expression, .5, and will throw away one figure in the answer for it.

$$\begin{array}{r} 287,375 \\ 6\$0 \\ \hline 22990000 \\ \$3,83,16,66\frac{2}{3} \end{array}$$

Here multiply by 8, annex the cipher, and divide by 6. The answer is 3 dollars, 83 cents, 16 hundredths, &c., &c.

$$\begin{array}{r} \$200 \\ \$15-5 \\ \hline .500 \end{array}$$

What is the interest on \$200, for 15 days? Ans. Fifty cents.

What is the interest on \$360, for 97 days?

$$\begin{array}{r|l} 360-0 & \\ 2 & 97 \\ \hline & \$15,820 \end{array}$$

We now come to questions in which the time is months, or months and days, or years, months and days. In this case 2 is used on the left, because in 2 months \$1 gains 1 cent interest; the answer will be in cents and hundredths, without throwing off one figure.

What is the interest on \$200, for 7 months? Here we place principal and time on the right, and 2 on the left;

$$\begin{array}{r|l} 200 & \\ 2 & 7 \\ \hline & \$7,00 \end{array}$$

the answer is \$7, and no cents.

Interest on \$387,20 cents, for 10 months? Two into 10 five times, and five times 387,20 are 19 dollars 36 cents, and no hundredths,

$$\begin{array}{r|l} 387,20 & \\ 2 & 10-5 \\ \hline & \$19,36,00 \end{array}$$

answer

What is the interest on \$47, for 5 months? In this instance, we multiply and divide only.

$$\begin{array}{r|l} 47 & \\ 2 & 5 \\ \hline & 235 \\ \hline & \$1,17\frac{1}{2} \end{array}$$

What is the interest on \$480, 93 $\frac{1}{4}$  cents, for 1 month? Three-fourths are made .75, in the form of two decimals: hence, in the answer, two figures are cut off for decimals, two for hundredths, and two for cents.

$$\begin{array}{r|l} 480,93,75 & \\ 2 & 1 \\ \hline & \$2,40,46,87\frac{1}{2} \end{array}$$

What is the interest on 12 $\frac{1}{2}$  cents, for 20 months? This principal is reduced to halves: answer, 1 cent and 25 hundredths; or 1 $\frac{1}{4}$  cents.

$$\begin{array}{r|l} 25 & \\ 2 & 20-5 \\ \hline & 1,25 \end{array}$$

$\begin{array}{r|l} \$80-2 & \\ \$13 & \\ \hline \$2,60 & \end{array}$ 
 What is the interest on \$80, for 6 months and 15 days? 15 days being  $\frac{1}{2}$  of a month, the time is  $6\frac{1}{2}$ , or  $1\frac{1}{2}$  months. Hence the statement.

$\begin{array}{r|l} \$1200-2 & \\ \$11 & \\ \hline \$22,00 & \end{array}$ 
 What is the interest on \$1200, for 3 months and 20 days? Twenty days are  $\frac{20}{30}$ , equal to  $\frac{2}{3}$  of a month:

hence the time  $3\frac{2}{3}$ , or  $1\frac{1}{3}$  months. In these instances, the denominators are placed on the left with the 2 months.

$\begin{array}{r|l} \$1000 & \\ \$57 & \\ \hline \$57,00 & \end{array}$ 
 What is the interest on \$1000, for 11 months and 12 days? Twelve days are  $\frac{12}{30}$ , equal to  $\frac{2}{5}$  of a month: the time, consequently, is  $11\frac{2}{5}$ , or  $2\frac{2}{5}$  months. Here, 5 times 2 on the left, equal 10, or cipher on the right. The answer is \$57,00.

What is the interest on \$1500, for three years, 8 months and 25 days? Three years and 8 months make 44 months: and 25 days are  $\frac{5}{6}$  of a month, which makes the time  $44\frac{5}{6}$  months. This reduced to an improper fraction, in  $2\frac{2}{3}$  months: thus,

$\begin{array}{r|l} \$1500-125 & \\ \$269 & \\ \hline \$336,25 & \end{array}$ 
 Twice six on the left, are 12, which goes into 1500, one hundred and twenty five times: and  $269 \times 125 = \$336,25$ , the answer.

What is the interest on 80 dollars, for 4 years, 10 months and 9 days?

$\begin{array}{r|l} \$80-4 & \\ 10 & 583 \\ \hline \$23,32 & \text{Ans.} \end{array}$ 
 Four years and 10 months, make 58 months, and 9 days make  $\frac{3}{4}$  of a month, which are  $58\frac{3}{4}$ , equal to  $7\frac{3}{4}$  months.

In cases where the time is an even number of years, it is only necessary to multiply

together principal, time and rate, and cut off 2 or 4 figures for cents, as indicated by the denomination of the principal.

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### SUMMARY OF DIRECTIONS.

*When the Time is DAYS, place the Principal and Time on the right, and 6 on the left. If the Principal is dollars, the answer is MILLS; here, cut off one figure at the right, for mills, and two more for cents: if the Principal is cents, the answer is thousandths of cents; here, cut off one figure for thousandths, two for hundredths, and two more for cents. Or, cut off one figure at the right in either case, and the answer will be cents, or hundredths, according to the denomination of the principal.*

*When the Time is MONTHS, place Principal and Time on the right, and 2 on the left. If the Principal is dollars, the answer is cents; if cents, the answer is hundredths of cents.*

*When the Time is YEARS, place Principal, Time and Rate on the right, and multiply continuously. The answer is cents or hundredths, according to the denomination of the principal.*

*When the Time is years and months, or years, months and days, reduce the years to months, and add all the given months: then reduce the days to the fractional part of a month, if practicable; annex this fraction to the months: reduce all to an improper fraction, and place the numerator on the right, and denominator on the left.*

*If the days cannot be reduced to the fractional part of a month, reduce the whole time, years, months, and days, or months and days, to days, and divide by 6, as in other cases.*

*When the principal is a mixed number, reduce it to an improper fraction, and place the numerator on the right, and denominator on the left: Or, express the fraction in decimals, and cut off as many figures on the right of the answer for decimals, as indicated by the number of decimals in the principal.*

#### INTEREST AT SEVEN AND EIGHT PER CENT.

The first method given in this book for reckoning at any rate per cent., is quite applicable to 7 and 8 per cent. Yet we may use a method which requires fewer figures, in connection with the 6 per cent. method just noticed.

In states where the rate of interest is 7 and 8 per cent., it will be found very convenient to ascertain the interest first at 6 per cent., by the short rule given, and then add  $\frac{1}{6}$  more for 7 per cent., or  $\frac{1}{3}$  for 8 per cent. One per cent. is the  $\frac{1}{6}$  of 6 per cent.: hence we may divide the interest at 6 per cent. by 6, and thus get  $\frac{1}{6}$ , which will be added to the former interest. So two per cent. over 6, making 8, is  $\frac{1}{3}$  of 6: hence the interest at 6 may be divided by 3, and the quotient added to the former interest.

When the rate is 12 per cent., instead of dividing by 6 as in 6 per cent. calculations, 3 only may be placed on the left of the line, and the answer divided off as in 6 per cent. calculations; because at this rate per cent. *one dol-*

lar in three days gives one mill interest. A few examples are given for illustration.

What is the interest on \$350 for 160 days at 7 per cent.? We work as in 6 per cent.; thus,

Placing 6 on the left, at 6 per cent. the answer would be \$9,33 $\frac{1}{2}$  cents. We now place 6 at the left of this answer, which is equivalent to multiplying the answer by  $\frac{1}{2}$ , not using the numerator on the right, and divide the first answer, setting the quotient beneath. This added gives for the answer, \$10,88 cents and 8 $\frac{1}{2}$  mills.

$$\begin{array}{r|l}
 350 & \\
 3-6 & 140-8 \\
 \hline
 3 & 28000 \\
 \hline
 6 & 9,33,3 \\
 & 1,55,5\frac{1}{2} \\
 \hline
 & \$10,88,8\frac{1}{2}
 \end{array}$$

Again: What is the interest on \$20 for 8 months at 7 per cent.?

Here, 2 is used on the left, the time being months, and the answer 80 cents is obtained. One-sixth or 13 $\frac{1}{2}$  added gives 93 $\frac{1}{2}$  cents. One advantage in this method is that the numbers to be divided by, on the left, are never large, and the fraction lost by the last division, is of no practical importance.

What is the interest on \$397,20 for 40 days at 7 per cent.?

$$\begin{array}{r|l}
 \$397,20 & -662 \\
 40 & \\
 \hline
 6 & 264800 \\
 & 44133\frac{1}{2} \\
 \hline
 & \$3,08,93,3\frac{1}{2}
 \end{array}$$

The answer in this instance is \$3 and nearly 9 cents.

What is the interest on \$180 for 371 days at 8 per cent.?

$$\begin{array}{r}
 180-3 \\
 6 \overline{) 371} \\
 \underline{3 \ 11} \ 130 \\
 \underline{3 \ 71} \ 0 \\
 \$14,84,0
 \end{array}$$

In this example, after finding the interest at 6 per cent., it is divided by 3, which is equivalent to multiplying by  $\frac{1}{3}$ , and the quotient is added, making 14 dollars and 84 cents, answer.

Again: What is the interest on \$399 for 20 days at 8 per cent.?

$$\begin{array}{r}
 399 \\
 8-8 \overline{) 20} \\
 \underline{3 \ 13} \ 30 \\
 \underline{4 \ 43} \ \frac{1}{2} \\
 \$1,77,3\frac{1}{2}
 \end{array}$$

The answer is one dollar, 77 cents, and  $3\frac{1}{2}$  mills.

What is the interest on \$487,20 for 4 months at 8 per cent.?

$$\begin{array}{r}
 487,20 \\
 8 \overline{) 4-2} \\
 \underline{8 \ 97} \ 440 \\
 \underline{3 \ 24} \ 80 \\
 \$12,99,20
 \end{array}$$

The time being months, we divide first by 2, and afterward, the answer thus obtained by 3.

What is the interest on \$1500 for 211 days at 12 per cent.?

$$\begin{array}{r}
 1500-5 \\
 12 \overline{) 211} \\
 \underline{1 \ 05} \ 50,0 \\
 \$105,50,0
 \end{array}$$

We here divide by 3 only, because, as said above, 1 dollar in 3 days at 12 per cent. gives 1 mill interest. Hence, one figure is cut off in the answer for mills.

When the time is months and the rate 12 per cent., the principal and time may simply be multiplied together; because 1 dollar in 1 month at 12 per cent., gives 1 cent interest. Hence nothing more is necessary than multiplication, as 1 placed on the left of the line could serve

no other purpose than to show the nature of the statement by proportion.

What is the interest on \$873 for 7 months at 12 per cent.?

The answer is in this, as in all other cases of months, obtained in cents, 61 | 873  
dollars and 11 cents. | 7  
\$61.11

DIRECTIONS FOR SEVEN, EIGHT, AND TWELVE PER CENTUM.

*State as in cases of 6 per cent.: if the rate is 7 per cent., divide the answer by 6, and add the quotient: if the rate is 8 per cent., divide the answer by 3, and add the quotient.*

*If the rate is 12 per cent., and the time days, place principal and time on the right and 3 on the left: cut off one figure at the right, and the answer will be cents or hundredths of cents.*

*When the time is in months, multiply principal and time together, and the answer will be in cents or hundredth of cents.*

*When the time is years, multiply principal, time, and rate together; and the answer will be cents, or hundredths of cents.*

RATE AND FORFEITURE TABLE.

RATES OF INTEREST legalized in the several states of our union, and in foreign countries, with the PENALTIES FOR USURY.\*

STATES.	RATES.†	PENALTIES FOR USURY.
Maine,	6 per cent.	Forfeit of entire debt.
New Hampshire,	6 " "	Three times the usury.
Vermont,	6 " "	Usury recoverable with costs.
Massachusetts,	6 " "	Three times the usury forfeited.

\* On all dues to the United States, 6 per cent. is charged, even where the states legalize higher rates.

† If the rate is not mentioned in the note, interest may be collected at the rate established by the law of the state in which the transaction occurs.

## RATE AND FORFEITURE TABLE.

STATES.	RATES.	PENALTIES FOR USURY.
R. Island,	6 per cent.	Forfeit of interest and usury.
Connecticut,	6 " "	Forfeit of entire debt.
New York,	7 " "	" " " "
New Jersey,	6 " "	" " " "
Pennsylvania,	6 " "	" " " "
Delaware,	6 " "	" " " "
Maryland,	6 " "	(1) Such contracts void.
Virginia,	6 " "	Forfeit twice the usury.
North Carolina,	6 " "	" " " "
South Carolina,	7 " "	Forfeit interest, usury, and costs.
Georgia,	8 " "	Forfeit three times the usury.
Alabama,	8 " "	Forfeit usury and interest.
Mississippi,	8 " "	(2) Forfeit of usury and costs.
Louisiana,	5 " "	(3) All such void.
Tennessee,	6 " "	" " "
Kentucky,	6 " "	Costs and usury recoverable.
Ohio,	6 " "	All such void.
Indiana,	6 " "	Forfeit of twice the usury.
Illinois,	6 " "	(4) Forfeit interest and three times usury
Missouri,	6 " "	(5) Forfeit interest and usury.
Michigan,	7 " "	Forfeit one-fourth debt and usury
Arkansas,	6 " "	(6) Whole usury forfeited.
Florida,	8 " "	Forfeit interest and usury.
Wisconsin,	7 " "	(7) Forfeit three times usury.
Iowa,	7 " "	(8) " " " "
Texas,	10 " "	All such void.
Dist. Columbia,	6 " "	" " "
England,	5 " "	Forfeit three times the debt.
France,	5 " "	
Ireland,	6 " "	
Canada,	6 " "	
Nova Scotia,	6 " "	
W. Indies,	8 " "	
Constantinople,	30 " "	

## MAKING AND TRANSFERRING NOTES.

In closing the article on interest, it may be well to give a few practical directions on making and transferring notes. Much litigation arises from inattention to the following requirements of law :

1. A promissory note is an instrument of writing in which the promisor or maker pledges the payment of money or property to a second person, at or before a specified time, in consideration of equivalent value received.

2. The sum of money or property for which the promisor gives the note, is called the "face of the note," and after being expressed in figures at the beginning of the note, should be written in words in the body of the same.

3. The words "value received" should always be found in the body of

(1) Contracts in tobacco may be as high as 8 per cent.

(2) By contract as high as 10 per cent.

(3) By agreement as high as 10; bank interest 6 per cent.

(4) By agreement as high as 12.

(5) By agreement as high as 10.

(6) Any rate not above 10 per cent.

(7) By contract as high as 12.

(8) By agreement as high as 12 per cent.

a note, as the laws require that money shall be paid only for a "consideration" or equivalent. Without this, notes are said to be invalid or worthless.

4. The individual who makes the note is called the drawer or giver; the person to whom it is given is called the payee; and the person who has it in legal possession, is called the holder.

5. The payee of a note may sell or transfer it to a third person, if it be written payable "to order," or "to bearer;" and this third person or holder may sue and collect as if he had been the original payee. Such a note is called negotiable, because it can be traded by the payee, and made payable to such person as he may order.

6. The law requires the holder of a negotiable note to indorse it, or write his name on the back, if he wish to sell it; provided that such note be transferable, or "payable to order." If the holder is unable to collect the note of the drawer, then the indorser is responsible, and can be made to pay it. The holder of a note which is made payable "to bearer," can transfer without indorsing it; and is, in this event, not liable for it. In the transfer of such notes there is said to be no recourse. Thus, if a bank-note is not indorsed by the individual who pays it, he is no longer liable to lose it, if it prove worthless.

7. A note made payable to a particular individual without the words payable "to order," or "to bearer," cannot be negotiated or traded to another; nor can another individual, except in the name of the payee, collect it.

8. A note specifying no time for payment, must be paid on demand. Such are generally called "due-bills."

9. Conventional usage, and in some instances law, has established, that a note shall not be collected until three days after it is due; and interest is calculated on these three days, as on the rest of the time. A note becoming due on Sunday, with these three days included, should be paid on Saturday. Three days thus given, is to allow for all exigencies; and are called "days of grace," because they are given gratuitously. Grace means gift.

10. Maturity of a note is the time specified for its payment. If a note is not paid at maturity that has been transferred, the holder must legally notify the indorser of the fact, or the indorser is released from his liability.

11. Interest cannot be charged on a note paid at maturity, without it has been specified; the words, "with interest," being inserted. But when a note not containing these words, is not paid until after maturity, the rate of interest legal in the state can be charged from maturity.

12. Notes bearing interest without the rate being inserted, bear the legal interest of the state in which drawn. Any agreement for a rate of interest less than the legal interest, must be specified, or legal interest can be collected.

13. Notes for any commodity of merchandise cannot be negotiated; and if payment is not made at the time specified, the holder may recover the amount in money. In such cases the three days grace are not allowed.

14. When two or more persons give a note conjointly, the payee or holder may collect it of either or any of them.

**PAYMENT ON NOTES.**—To compute interest on notes, we must ascertain the time which elapses between the period when interest commences, and

that on which the payment is made, by subtracting the former from the latter date,\* which is done thus:

What is the interest on a note for \$500, dated May 20, 1843, and paid January 19, 1845?

yrs. mos. days.

1845 " 1 " 19

1843 " 5 " 20

Time, 1 " 7 " 29

After arranging the former time under the latter thus, if the number of days in the lower line is larger than that in the upper, 30 days must be added to the upper line, and the subtraction made from the whole number above, and the remainder set under the days. One is carried to the lower line of months. If this number of

months is larger than that above, 12 must be added above and the subtraction continued as before. It will be observed here, that the months are placed down according to the order they occupy in the year. May is the 5th month; hence we use 5 as the number; so is 1 used for January, it being the first month.

#### PARTIAL PAYMENTS.

Below, we give the Ohio rule for casting the interest on partial payments, which is the method used in Indiana, Kentucky, and most of the states of the union.

The rule, and the calculation to illustrate its application, are extracted from *Swan's Treatise*.

" The Ohio rule for calculating partial payments, is as follows: Where payments exceeding the interest are made after the debt is due: In such case interest should be calculated on the debt up to the time of payment, and the principal and interest then added together, and the payment subtracted from the total. Subsequent interest should be computed on the balance of principal thus found to be due.

" Where the payment is less than the interest due, the surplus of interest must not be added to the principal; but interest continues on the former principal, the same as if no

\* The day on which a note is dated, and that on which it becomes due, should not both be reckoned. The former is excluded among business men.

payment had been made, until the period when the payments added together, exceed the interest due; and then the surplus of payments is to be applied towards discharging the interest. For instance, upon a note for \$100, payable in one year with interest, if a payment of 10 dollars is made at the end of two years, and 10 dollars at the end of four years, and 19 dollars at the expiration of six years; here interest on the whole amount of the note should be calculated up to the time of the payment of the 19 dollars, and then the several payments should be added together, and deducted from the amount of all the principal and interest; the balance would be the amount due, and upon which interest should be afterwards computed.

The following calculations will illustrate the rule in the text:—A., by his note, dated Jan. 1st, 1840, promises to pay to B. 1000 dollars, in 6 months from date, with interest from the date. On this note are the following endorsements: Received, April 1, 1840, 24 dollars; Aug. 1, 1840, 4 dollars; Dec. 1, 1840, 6 dollars; Feb. 1, 1841, 60 dollars; July 1, 1841, 40 dollars; June 1, 1844, 300 dollars; Sept. 1, 1844, 12 dollars; Jan. 1, 1845, 15 dollars; Oct. 1, 1845, 50 dollars; and the judgment is to be entered Dec. 1, 1850.

## CALCULATION.

The principal sum carrying interest from January 1, 1840,	\$1000 00
Interest to April 1, 1840, 3 months,	15 00
Amount,	1015 00
Paid April 1, 1840, a sum exceeding the interest,	24 00
Remainder for a new principal,	991 00
Interest on \$991 from April 1, 1840, to February 1, 1841, 10 months,	49 55
Amount,	1040 55
Paid August 1, 1840, a sum less than the interest due,	\$4 00
Paid December 1, 1840, do. do.	6 00
Paid February 1, 1841, do. greater do.	60 00
	70 00
Remainder for a new principal,	970 55
Interest on \$970 55 from February 1, 1841, to July 1, 1841, 5 months,	24 26
Amount,	994 81
Paid July 1, 1841, a sum exceeding the interest,	40 00
Remainder for a new principal,	954 81
Interest on \$954 81 from July 1, 1841, to June 1, 1844, 2 years 11 months,	167 00
Amount,	1121 81
Paid June 1, 1844, a sum exceeding the interest,	300 00
Remainder for new principal,	821 81
Interest on \$821 81 from June 1, 1844, to October 1, 1845, 1 year and 4 months,	65 75
Amount,	887 56
Paid September 1, 1844, a sum less than the interest due,	\$12 00
Paid January 1, 1845, do. do.	15 00
Paid October 1, 1845, do. greater, with the two last payments, than the interest then due,	50 00
	77 00
Remainder for new principal,	810 56
Interest on \$810 56, from October 1, 1845, to December 1, 1850, the time when judgment is to be entered, 5 years and 2 months,	251 30
Judgment rendered for the amount,	\$1061 86

The following is the rule of the Supreme Court of the United States, as given by Chancellor Kent, Johnson's Chancery Reports, vol. 1st, page 17; and is adopted by most of the States of the Union, among which are Massachusetts and New York:

## SUPREME COURT RULE.

I. *"The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due."*

II. *"If the payment exceeds the interest, the surplus goes toward discharging the principal; and the subsequent interest is to be computed on the balance of the principal remaining due."*

III. *"If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal; and interest is to be computed on the balance as aforesaid."*

A. gave to B. his note for 12,000 dollars; at the expiration of three months he paid 2,000 dollars; in three months more 6,000, and at the expiration of three months more, 3,000 dollars: what did he pay to B. when the note was taken up at the close of the year, the note being made on the 1st day of January? We reckon these payments by the Supreme Court Rule.

Principal - - - - -	\$12,000
Interest on the whole for three months - - -	180
Amount of principal and interest - - - - -	12,180
First payment to be deducted - - - - -	2,000
Balance due after first payment - - - - -	10,180
Interest from 1st to 2d payment, 3 months - -	152.70
Amount to be reduced by 2d payment - - -	10,332.70
Second payment to be deducted - - - - -	6,000.
Balance due after 2d payment - - - - -	4,332.70
Interest from 2d to 3d payment, 3 months - -	64.99.05
Amount to be reduced by 3d payment - - -	4,397.69.05
Third payment to be deducted - - - - -	3,000.
Balance due after 3d payment - - - - -	1,397.69.05
Int. from 3d pay't till settlement, 3 months	20.96.53.57½
Balance due on settlement - - - - -	1,418.65.58.57½

The following is called the Commercial Rule, and is adopted by Vermont:

COMMERCIAL OR VERMONT RULE.

*Find the amount of the whole debt until the time of settlement; then find the amount of each payment from the time of payment until the time of settlement; add these, and subtract the sum from the former amount: the remainder will be the sum due.*

For all payments made within one year, this rule is identical with that of Connecticut, which follows:

CONNECTICUT RULE.

I. "Compute the interest on the principal to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, until all the payments are absorbed; provided the time between one payment and another be one year or more

II. "If any payments be made before one year's interest has accrued, then compute the interest on the principal sum due on the obligation, for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.

III. "If a year extends beyond the time of payment, then find the amount of the principal remaining unpaid up to the time of settlement, likewise the amount of the indorsements from the time they were paid to the time of settlement, and deduct the sum of these several amounts from the amount of the principal.

"If any payments be made of a sum less than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period."

KIRBY'S REPORTS.

Ignorance of the correct method of calculating interest on partial payments, is the cause of much litigation. Hence, it behoves men to remember, that interest should be reckoned till the time of the first payment, and added to the principal, and the payment deducted; provided the payment is greater than the interest that has accrued. But if the interest is greater than the payment, the payment must be set aside, and the interest reckoned to another payment; or continuously from one payment to another, till the sum of the payments shall exceed the sum of the interest accrued. Then the several sums of interest should be added to the principal, and the sum of the several payments deducted. The remainder will be a new principal on which interest runs till the next current payment, or till the debt is paid, or judgment rendered. This method will stand in the courts, of the great majority of the States in the Union.

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## RATIO AND PROPORTION.

That the principles of subsequent statements may be understood, we introduce a few remarks on Ratio and Proportion. The principles and various ramifications of this subdivision of numbers, could not be well developed and elucidated, in less space than is occupied by this entire volume. Hence, we attempt only its outline.

*Ratio is the relation that exists between different*

*quantities and numbers of similar things*; and is always expressed by an abstract number. Ten apples are twice as many as five apples: hence, the *ratio* or *relation* is 2. We have here two different numbers of the same object: but it could not be said that, ten apples were twice as many as five biscuits; because there is no relation between one apple and one biscuit: neither can one be said to be larger than the other. Again, 15 yards of cloth are five times as many as 3 yards of cloth: here the ratio is 5. Twenty gallons of melasses are 4 times as many as 5 gallons: but 20 gallons of melasses are not 4 times as many as 5 gallons of rum.

Six bushels wheat are one fifth times as many as 30 bushels: that is, the ratio is  $\frac{1}{5}$ . Hence, the fact assumed in the outset is proven, that Ratio is the relation between *similar* things.

Two ratios make a proportion; that is, as many times greater or less as a *second* thing is than the *first*, so many times greater or less is a *fourth* than the *third*. These relations are ascertained, by first ascertaining the ratio that exists between the two numbers or quantities in the proposition, which are alike. For instance: If 4 yards of cloth cost \$5, what will 12 yards of cloth cost? Here the ratio between the 4 and 12 yards must be obtained first. This is done by using a vertical line, which is considered the fulcrum of a scale or balance, thus,

The 12 yards are placed on the right,  
 $\begin{array}{c} 4 \mid 12 \\ \hline \end{array}$  and the 4 yards *opposite*, on the left.  
 Hence, as in the two ends of the scale, we compare their value. Four is contained

in 12 three times: hence, the ratio is 3. Now, the four yards on the left, are equal in value to 5 dollars; which to balance the 4 yards, must be placed, likewise, *last* on the right. The ratio being an abstract number, and being on the right with the dollars, may be multiplied into the dollars: hence, 3 times \$5 are 15 dollars; not 15 bushels, or 15 of any thing else, except the denomination into which the abstract ratio is multiplied.

From this, the philosophy of the statement seems to require, that the price or denomination of the answer, be placed last, on the right, where it can be multiplied by the ratio between the two quantities above, and increased or decreased accordingly, as the ratio is an integer or fraction. From this, too, to keep up the theory of equilibrium, that which in value or extent, equals the price or name of answer, must be placed opposite, or on the left: then the number which is to be compared with this supposed quantity on the left, must be placed on the right, directly opposite the term on the left, for the purpose of ascertaining the ratio. When stated thus, it is not essential that the ratio be *actually* obtained, by dividing one upper term into the other; for the position indicates the ratio, and the numbers may be canceled by using one of the numbers with either of the other two.

In applying this statement to questions in Profit and Loss, the conditions of the question must be strictly noticed: cost price, compared with cost price; par per cent., with par per cent.; reduced with reduced, and advanced

with advanced per centum. By this means, and the directions following, the statements made in Profit and Loss, become rational and easy.

In questions where any of the terms are fractional, or mixed numbers, such mixed numbers may be reduced to improper fractions, and their numerators located in such position on the line, as otherwise the whole number would occupy; with all respective denominators opposite.

If  $7\frac{1}{2}$  yards cloth are worth \$20, what will  $18\frac{1}{2}$  yards come to? The term of demand, or which is *connected* with the demand, is  $18\frac{1}{2}$  yards. This must be placed on the right of the line first, in the form of  $7\frac{1}{2}$ . The numerator 75 will consequently occupy the right, thus,

$\begin{array}{r l} 75 & 5 \\ \hline 15 & 2 \\ \hline 150 & \end{array}$	The $7\frac{1}{2}$ belongs to the left, and being made $\frac{15}{2}$ , we place the numerator 15 on the left: now the term in which the answer is required, \$20, is placed last on the right: so
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that being multiplied by the ratio between the two quantities above, it will be made more or less. The question has now lost its fractional form; and being easily stated, can be *more* easily wrought. Thus, if the terms were such that they could not be canceled, they might be multiplied on each side; and the right divided by the left, with very little trouble.

If  $\frac{1}{3}$  of 6 yards cost \$3, what will  $\frac{1}{4}$  of 20 yards cost? Here,  $\frac{1}{4}$  of 20 is the demand, and is placed on the right, by its numerators, 1 and 20:  $\frac{1}{3}$  of 6 is the supposition of the same

name, and is placed opposite, by its numerators: the price, \$3, is placed last on the right; thus,

This may be proven by saying, if  $\frac{1}{4}$  of 20 cost  $7\frac{1}{4}$ , what will  $\frac{1}{4}$  of 6 cost?  $\frac{1}{4}$  of 6 is the demand;  $\frac{1}{4}$  of 20 the same name, and  $\$7\frac{1}{4}$  the term of answer, all of which are located accordingly, by their numerators; thus,

$$\begin{array}{r|l} 4 & 1 \\ 20 & 5 \\ 1 & 2 \\ 2 & 6 \\ \hline & 3 \\ \hline & \$7\frac{1}{4} \end{array}$$

The result is three dollars, and the question is proven.

$$\begin{array}{r|l} 3 & 1 \\ 1 & 6 \\ 20 & 4 \\ \hline & 215 \\ \hline & \$3 \end{array}$$

This department of numbers is called "the rule of three," because three terms are given and used in a statement to find a fourth. When this fourth term is ascertained, the proposition is complete; so, also, is the proportion.

The rule of three having three terms given to find a fourth, we conclude that, two of these being either *means* or *extremes*, we may divide either couplet by the remaining mean or extreme, and find such mean or extreme as constitutes the fourth term.

*It is a doctrine in proportion, that the product of the two means equals the product of the two extremes.* If this is true, it is evident that the product of either the means or extremes, is the result of the multiplication of the two terms together as factors.

Suppose the extremes be multiplied and the product divided by one of the means; it is evident that the other mean will be obtained, and

that if it be multiplied with the first mean, their product will equal that of the extremes. Or, if the product of the means be divided by one of the extremes, the other extreme will be obtained, which multiplied with the given extreme will yield a product equal to that of the means.

In the proportion  $2 : 4 :: 8 : 16$  (two to four as eight to sixteen), we multiply 8 by 4, making 32; this divided by 2, one of the extremes, gives 16, the other extreme: or, divided by 16, gives the extreme 2. If, again, the two extremes, 2 and 16, be multiplied, and the product, 32, divided by 4, one of the means, the other mean, 8, is obtained: or, if this product be divided by 8, the other mean, 4, is obtained. This law will be found necessary in the explanation of Compound Proportion, by cause and effect.

It will require care and attention in the learner to distinguish the demand at times, the language of questions being frequently so transposed as to present seemingly new issues to those not acquainted with, or accustomed to, analyzing questions before stating them.

The general nature of every question must be understood before much can be done by any process of solution. And this is the only service that analysis can render, *the elucidation of the general bearings of the problem.*

We have always been opposed to the mechanical routine by which persons have gone on from one question to another, and wrought some, merely because others were wrought so. So long as this method of solving questions

by a *rule*, whose principles may at first have been understood, is practiced among those seeking a first knowledge of the science, we may not look for such a knowledge of the subject as will enable them to apply the principles to the great every-day transactions of life.

We shall now solve a variety of problems and prove them; that after the path is pointed out, the pupil may pursue it with some degree of pleasure.

If 10 yards of cloth cost \$16, what will 15 yards cost?

The demand, 15 yards, is placed on the right, and the term of the same name, 10 yards, opposite; while the price, 16 dollars, is placed last on the right, where it can through multiplication, be increased by the ratio of the two numbers of yards. We use the factor 5 into 10 and 15. The answer is 24 dollars.

We prove this by saying, if 15 yards cost 24 dollars, what will 10 cost?

Thus it is proven that 10 yards cost the 16 dollars assumed in the outset.

We may now state yet differently, and say, if \$16 buy 10 yards, how many yards will \$24 buy? thus,

Here 24 dollars become the demand, and 16 the *same name*; while 10 yards is the denomination of answer, and is consequently placed on the right. The ratio is ob-

$$\begin{array}{r|l} \$-16 & 15-3 \\ \hline & 16-8 \\ & \$24 \end{array}$$

$$\begin{array}{r|l} \$-16 & 10-2 \\ \hline & 24-8 \\ & \$16 \end{array}$$

$$\begin{array}{r|l} \$-16 & 24-3 \\ \hline & 16-5 \\ & 15 \text{ yds} \end{array}$$

tained between the dollars, and multiplied into the 10 yards, increasing them to 15 yards.

$$\begin{array}{r} 24 \text{ } 16 \text{ } 2 \\ 15 \text{ } 5 \\ \hline 10 \text{ yds.} \end{array}$$

Again: if \$24 buy 15 yards, how many will 16 buy?

The first form of the question above would be stated and solved thus by the old method:  $10 : 15 :: 16 : x$ . The *second* and *third* terms, 15 and 16, would be multiplied together and their product divided by the first; thus,

$$\begin{array}{r} 10 : 15 :: 16 : x \\ 15 \\ \hline 80 \\ 16 \\ \hline 10)240 \\ 24 \text{ dolls.} \end{array}$$

By this mode of proceeding, the figures are accumulated to a large number by first multiplying up, and then dividing down again; whereas both operations might be dispensed with, and the term

of answer multiplied simply by the ratio between the 10 and 15.

It appears singular that any intelligent author should teach that it was necessary in such a question as the one just wrought, to multiply the 16 dollars by the 15 yards, which is within itself impossible; for such an operation has no meaning. Fifteen yards times 16 dollars will make neither 240 yards nor 240 dollars. It may be urged that it is multiplying the dollars by the abstract terms of a fraction, which represents the ratio. This may be admitted in the case of those prime numbers whose ratio cannot be reduced to a single expression. Even then we would prefer expressing it by a positive fraction, in a fraction's

form, or by a mixed number, either of which would indicate how many times the dollars, or other denomination of answer, was to be taken. For when the other process is pursued, the whole idea of *ratio* between the number multiplied and divided by, is lost; and the pupil multiplies and divides just because the rule so teaches him. But tell him to get first the ratio between two numbers, or quantities, and then to multiply the denomination in which the answer is required, by it, thus increasing or decreasing it, and he understands what he is doing. He knows that if the ratio be larger than 1, he will have an answer as much larger than the last term as this ratio is greater; and if the *ratio* be smaller than unity, his answer will be smaller than the term to be multiplied by such ratio. But that the second and third shall be multiplied together to a product, but to be divided down again by the first, is an absurdity that in its very form excludes the idea that we should keep constantly before us, that of *ratio*. It is not that the boy could not understand ratio, but that the mode of applying it had so little affinity to the principle, that when the application was commenced in multiplying and dividing, he found himself in a new field without any discernable directions to his point of destination. Such absurd regulations give the young an idea that nothing can or must be attempted beyond the comprehension of mere mechanical landmarks; while if he were told that he must get the ratio between the two numbers, and increase or decrease another by

it, the mist would flow from his eyes, and develop the sunlight of unclouded truth.

If 5 sheets of paper make 150 pages of a book, how many sheets are required to make 800 pages?

$$\begin{array}{r|l} 3 & 150 \\ \hline & 800 \end{array}$$

It requires  $26\frac{2}{3}$  sheets.

If  $26\frac{2}{3}$  sheets make 800 pages, how many will 5 sheets make?

$$\begin{array}{r|l} 5 & 800 \\ \hline 3 & 150 \end{array}$$

In this instance the demand is 5 sheets, and the same name  $26\frac{2}{3}$  sheets; the latter is placed on the left after being reduced to  $\frac{80}{3}$ . That is, the numerator is

placed on the left, where the term of *same name* should go, and 3, the denominator on the right. Knowing that  $26\frac{2}{3}$  sheets make 800 copies, and that in the use of this 800 we must find how many five sheets will make, we place the 800 last on the right, that the answer may be in pages.

If 240 feet of lumber cost 9 dollars, how many feet can be purchased at  $18\frac{1}{2}$  dollars?

$$\begin{array}{r|l} 18\frac{1}{2} & 240 \\ \hline 9 & 500 \end{array}$$

The two terms to be compared are dollars: hence  $18\frac{1}{2}$  dollars is the demand. The answer is the number of feet

which this sum buys, 500 feet.

If a chicken that cost 5 cents is sold for 8 cents, what is the gain per 100 cents?

$$\begin{array}{r|l} 8 & 100 \\ \hline 5 & 60 \end{array}$$

We know that 5 cents gains 3 cents, and inquire, what will 100 cents or per centum gain? The answer is 60 per cent.

If a perpendicular wall 80 feet high cast a shadow at noon 60 feet wide, how wide a shadow will a perpendicular church steeple cast, which is 240 feet high?

Here the terms to be compared are not the wall and the steeple, which are both perpendicular, but the separate *heights* of these two dissimilar objects. The answer is desired in width or extent; this is consequently placed last on the right, where it can be multiplied by the ratio between the different heights. Answer, 180 feet wide.

$$\begin{array}{r} 80 \overline{) 240} - 3 \\ 60 \\ \hline 180 \end{array}$$

If a shadow 180 feet wide be cast by a steeple 240 feet high, how high must the steeple or other perpendicular object be that will cast a shadow 60 feet wide?

Here the widths are compared, and the answer is obtained in height.

$$\begin{array}{r} 2 - 180 \overline{) 60} \\ 240 - 8 \\ \hline 80 \end{array}$$

How many gallons of oxygen will be necessary to make 720 gallons of water, if 9 gallons of water require 8 gallons of oxygen?

We compare water with water. The answer is 640 gallons oxygen.

$$\begin{array}{r} 8 \overline{) 720} - 8 \\ 8 \\ \hline 640 \end{array}$$

If nine gallons of water require 1 gallon of hydrogen, how much hydrogen is required to make 720 gallons of water?

The answer is 80 gallons of hydrogen.

$$\begin{array}{r} 8 \overline{) 720} - 8 \\ 1 \\ \hline 80 \end{array}$$

If after I see the flash of a cannon, I hear.

the report in 4 minutes, how far will it be off, if sound flies at the rate of 1142 feet per second?

$$\begin{array}{r}
 1 \overline{) 240} \\
 8 \overline{) 1142} \\
 1760 \overline{) 1} \\
 \underline{1} \\
 51 \frac{1}{2}
 \end{array}$$

Four minutes make 240 seconds, which is the demand, while 1 second is the same name, and 1142 feet, the distance which sound flies in the one second, is the term of answer. We say, how many yards will these feet make, if 3 feet opposite make 1 yard: and, again how many miles will these yards make if 1760 yards opposite make one mile? Thus three distinct proportions are combined in one, which produces no inconsiderable economy in the use of figures.

If  $\frac{2}{3}$  of a pound of butter costs  $4\frac{1}{2}$  cents, what will  $1\frac{1}{2}$  pounds cost?

$$\begin{array}{r}
 2 \overline{) 3} \\
 2 \overline{) 3} \\
 4 \overline{) 19} \\
 16 \overline{) 171} \\
 \underline{10 \frac{1}{2}}
 \end{array}$$

The demand is placed on the right as  $\frac{2}{3}$ , the same name on the left as  $\frac{2}{3}$ , and the price last on right as  $\frac{1}{2}$ . It is seen that the numerator of the *same name* occupies the left, and its denominator the right.

None of the numbers in this question can be canceled; yet no sane man will dispute that the question is stated in much better form than by the old rules; for here the pupil sees at a glance what terms must be multiplied together, and what divided by, from their very location on the two sides of the line; which is not the case in the old method. Then, the statement being rational and easy, it is far preferable, though not a figure can be canceled.

If  $4\frac{1}{2}$  pounds of wool cost 30 cents, what will  $18\frac{1}{2}$  pounds come to?

This example may easily be proven by using the answer, in making inquiry with regard to some other portion of the question.

$$\begin{array}{r|l}
 2-4 & 25 \\
 2-8 & 5 \\
 \hline
 & 125
 \end{array}$$

“A hare starts 12 rods before a hunter, and scuds away at the rate of 10 miles an hour: now, if the hunter does not change his place, how far will the hare get before him in 45 seconds?”

The demand is here 45 seconds, which we place on the right: now, that we may reduce the hours to seconds and use them on the left, we place opposite the 45, 60 seconds, which make a minute; while the one minute is placed on the right last, as the denomination of answer. But that we may continue on, and reduce these minutes to hours, we place opposite this one minute 60 minutes, which make an hour, and which is equivalent to 10 miles running of the hare. As this 60 minutes equals the 10 miles running, we must place the latter last on the right; for we wish the answer in distance. Now, it is evident that if the question were wrought without going farther, the answer would be in miles; but wishing it in rods, we place 1 mile opposite 10, and 8 furlongs on the right, saying, how many furlongs will all the miles on the right make, if 1 mile makes 8 furlongs? We say again, how many rods will all of these furlongs make, if 1 furlong opposite the 8, make 40 rods; which being the term of answer, is placed last on the right; thus,

<del>2</del>	<del>60</del>	<del>45</del>	<del>15</del>	<del>5</del>
<del>2</del>	<del>60</del>	1		
	1	10		
	1	8		
		<del>40</del>		
		40		
		12		
		<hr/>		
		52 rods.		

The 12 rods which the hare had in the start, added to this, makes the answer, 52 rods.

Again: "If a dog by running 16 miles an hour gain on a hare 6 miles every hour, how long will it take him to overtake her, if she has 52 rods the start?"

40	52	The demand is 52 rods, and 40 rods
8	1	opposite equal 1 furlong, 8 furlongs
6	1	equal 1 mile, 6 miles equal 60 minutes
1	60	of running, and 1 minute equals 60
	60	seconds: hence the answer is 97½
	<hr/>	seconds.
	97½	

Again: "A hare starts 12 rods before a greyhound, but is not perceived by him until she has been up 45 seconds; she scuds away at the rate of 10 miles an hour, and the dog after her at the rate of 16 miles an hour: what space will the dog run before he overtakes her?"

2	195	What will 97½ or 1½ <sup>s</sup> seconds be, if
60	1	60 seconds make 1 minute, and 60
60	16	minutes equal 16 miles running of
1	8	the dog, and 1 mile equal 8 furlongs,
1	40	and 1 furlong equal 40 rods, the de-
	<hr/>	nomination of the answer? The dog
	138½	will run 138½ rods.

These questions cannot be fully elucidated

in this little work, and are given merely to indicate the capabilities of this beautiful system of statement and solution.

If  $\frac{7}{5}$  of a pound of rice feed 3 men, how many will  $\frac{21}{10}$  pounds feed?

We treat complex fractions in the statement as all others, placing the demand on the right, same name on the left, etc. In the demand, the numerator of the numerator is placed on the right, and the numerator of the denominator on the left, with all respective denominators opposite their numerators.

$$\begin{array}{r|l}
 2\frac{1}{5} & \\
 3-10 & 2\frac{1}{5} \\
 7 & 3-2 \\
 & 5 \\
 & 3 \\
 \hline
 & 7\ 30 \\
 \hline
 & 4\frac{2}{5}
 \end{array}$$

If  $\frac{41}{15}$  of 2 yards of muslin be worth  $\frac{11}{81}$  yards of gold lace, how many yards of gold lace will pay for  $\frac{40}{3}$  of 12 yards of muslin?

Five times 3 equal 15; 3 into 9 three times, and 3 times 2 equal 6; 5 into 10 twice, and into 25 five times.  $4 \times 4 \times 2 \times 2 \times 12 = 768$ , which divided by 5 gives  $153\frac{3}{5}$  yards of gold lace for the answer.

$$\begin{array}{r|l}
 2\frac{1}{5} & 40-2 \\
 4 & 4 \\
 12 & 12 \\
 3-2 & 2 \\
 15 & 15 \\
 5-25 & 5 \\
 & 5 \\
 \hline
 & 5\ 768 \\
 \hline
 & 153\frac{3}{5}
 \end{array}$$

All such propositions as these, though not practical in their bearing, will, nevertheless,

afford interesting entertainment to those studying for the mere beauty of *theory*, while it is believed that a sufficient number of examples have been given to meet all practical purposes.

We are aware that many authors consider the treatise of Proportion under two heads, as superfluous. It may be superfluous when treated mechanically; but when cause and effect, as the bases of these principles, are developed, they induce the division on natural, rational, and necessary grounds.

#### SUMMARY OF DIRECTIONS FOR DIRECT PROPORTION.

*Ascertain first the term of Demand:*

*Place the Demand first on the right:*

*Place the term of the Same Name opposite the Demand, on the left:*

*Place the term in which the answer is required, last on the right, and the answer will be in the same denomination.*

*In all fractional terms, the Numerator must occupy the side of the line ordinarily assigned to the integer.*

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#### PROFIT AND LOSS.

Profit and Loss, as constituting a department of the Arithmetic, relate to the various gains and losses of general business transactions. They are reckoned in two ways: *spe-*

*cifically*, and at a *certain per centum*. A *specific* gain, is where any gross quantity, at a given cost price, is sold at some other price, without reference to any regular sum of profit: as 30 galls. of wine, which cost \$19, being sold at \$31,40; hence, the gain is \$12,40, on the cost.

Gain or loss per cent., is where a certain sum is gained on the hundred; for instance, a bill of goods costs \$50; but it is desired to sell it at 30 per cent. profit; consequently, it must be sold at \$65; the 15 dollars gained, being a systematic profit.

Some business men adopt a conscientious mode of trading, and are willing to gain such a per cent. profit, as is just and reasonable; while others, of less principle, and more exorbitant in their exactions, make the ignorance, credulity, or necessity of the purchaser, their only standard; and grasp any advantage that circumstances may afford them.

Some individuals calculate their profits and losses with reference to time; while others, and indeed the great mass of business men, regard only the simple transaction. That we should reckon with regard to the length of time occurring between the transactions of buying and selling, appears very reasonable, when we consider that, *all capital, as a medium of gain, has delegated to it, all the productive capacities of active individual effort.*

The general and unavoidable expenses of every establishment, require the influx of a constant gain to preserve the capital stock entire. Hence, the capital stock that a man invests in business, must yield periodically, and

at short intervals, a sufficient amount of profit to sustain these. And if the capital, which may be the only gain producing element in a man's business, be permitted to lie during a long interval, between the purchase and sale, the profit on the sale when made, must be larger in a degree corresponding to the length of time thus invested; or the deficit in meeting the expenses, must convert this seeming gain into a positive loss.

Let us suppose money worth 10 per cent. at interest. The business man who has 2000 dollars may easily lend it, and realize at the end of the year, 200 dollars profit. But he prefers investing it in goods, which he will sell at 10 per cent. profit. If the stock of goods be sold in 6 months, he may invest again, and sell again, in 6 months. Thus he would make 400 dollars clear money. But suppose he sell his first stock at the end of the year, at the 10 per cent. profit; he will gain 200 dollars. Suppose, again, he sells only half, it is manifest that he gains only 100 dollars; or that, if he sell the whole in a year and a half, his gain will be 200 dollars in  $1\frac{1}{2}$  years, or  $6\frac{2}{3}$  per cent. per annum.

Profit and Loss may be divided into five distinct varieties. They are,

*First:* To find how an article must be sold to gain or lose a certain per centum; or to find the sum of gain or loss, at a specified per centum.

*Second:* To find the rate per cent., profit or loss, when an article is purchased at one price and sold at another.

## VARIETIES OF . . .

*Third:* To find the cost price, when <sup>an</sup> article has been sold at a certain per cent., gain or loss.

*Fourth:* To find the rate per cent., gain or loss, when an article, sold at a certain price, with a specified gain or loss, is advanced or reduced to yet another price.

*Fifth:* To find the selling price of an article, whose cost price is affected by commission, premium, discount, loss or drawback, gain or loss per cent., &c.

All operations coming under any of these five heads in Profit and Loss, depend primarily on the principles of ratio and proportion. Operations in specific profit and loss, depend on addition and subtraction, only. Per centum being the great acknowledged basis in the rule, everything is in a ratio, greater or less than 100. The arbitrary rules in all of the old books, on this subject, have tended to make this simple and beautiful department of Arithmetic, complex; and even, in many cases, unintelligible; whereas, when the relations and bearings of the proportional principles involved, are demonstrated, we see a harmony and system, a regularity and order, that no other portion of the science can excel. Calculations in profit and loss are, however, of a more apparently abstract nature, that in the general calculations of ordinary business concerns. Hence, it becomes necessary, that we make nicer distinctions between the terms, and their specific names and qualities, than where material objects, such as yards, lbs., &c., are concerned. Here, we have to compare cost

### RAINEY'S IMPROVED ABACUS.

price with cost price, selling price with selling price, advanced with advanced, and reduced with reduced price. And, too, these distinctions are vital and necessary; for they constitute supposition, demand, and term of answer. Some authors work profit and loss by a system of decimals, which, although correct, yet obscures entirely the principles of ratio; making the statement, as well as the work, depend on a mechanical arrangement.

#### VARIETY FIRST.

If a lb. of coffee cost 10 cents, for how much must it be sold to gain 20 per cent.? We may remark here, that the cost price of an article is called the "*par*" price; *par*, as meaning the equal of something else, or the equal of the price. When, therefore, we say the *par* of per centum, we mean the 100, without increase or decrease, as its own established rate indicates. When we say the *par* price of an article, we mean the cost. Thus *par* price, and *par* per centum, may be considered the same thing. When 100 is compared with any specific thing, as cents, in the above case, it loses its abstract, and assumes the denominate form of the specific thing with which compared. Hence, in the case above, we may say that the 100 becomes 100 *cents*, because compared with 10 cents. Now, the position assumed at first was, that per centum, or 100, gained 20, which, changed from this abstract form to the denominate, is 100 cents giving 20 cents, or 100 cents being advanced to 120 cents; for, if 100 cents,

or anything else, gain 20, they must be advanced to 120.

Now, we know that 100 is the par, or cost, or first value of per centum; but 120 is the advanced price or value of it. Ten, we know, likewise, is the price of the coffee. Now, our demand, 10 cents, cost price, is placed on the right; 100, or per cent., opposite, on the left; and 120, advanced price, last on the right; consequently, the answer would be in the advanced or selling price of the coffee. Therefore, the ratio is obtained between the 10 cents and the 100 cents; and the selling price, 120, is multiplied by it. The ratio is  $\frac{1}{10}$ ; consequently,  $\frac{1}{10}$  of 120, or 12, is the selling price of the coffee, thus:

We may, instead of placing the advanced or selling price last on the right, place the gain of 100 there; get the answer, in gain, and add it to the cost price. If 100 gain 20, what will 10 gain? We add this gain, 2, to the cost price, and have, as before, 12, selling price. The latter process, however, is no more perspicuous, while it is more tedious than the former. Hence, it is best to find the selling price, by placing the advanced or reduced per cent., last on the right. Let us now find how we will sell this 1 lb. to lose 20 per cent. We know that if per cent. loses 20, it will be reduced to 80; so, if 100 cost, be 80, selling price, what will be the selling price of 10, cost?

Let us find the result, by the 2d process, as

before ; by ascertaining first, the loss, and then subtracting it from the 10.

$\begin{array}{r} 100   10 \\ \quad   20 \\ \hline \quad   2 \end{array}$	What will 10 lose, if 100 lose 20 ? We have 2 as before, which subtracted, leaves 8.
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One thing may be noticed very particularly, in the above ; that the *sum of profit or loss on any given sum, as the 100 above, is the same.* In this case 10 either *gained*, or *lost* 2. This leads us to remark, that a large sum will gain or lose more than a smaller. While 10 either gains or loses 2, 12 will gain or lose more than 2: 20 cents will gain or lose twice as much as 10 cents, while 5 cents will gain or lose only one half as much. We make these remarks, because there are some who may not apprehend the difference between the gains or losses on larger or smaller sums. Such think that if 20 per cent. gain will advance 10 to 12, as a matter of course, 20 per cent. *loss* will reduce 12 to the 10. The difference consists in this, that 10 will gain 2, and advance to 12: but 12, losing more than 2, will be reduced to  $9\frac{1}{2}$ . It will be well for the pupil to bear this difference in mind, as its importance will be seen in the calculations that follow. We might prove the foregoing questions, by asking the cost price, after knowing the advanced price, 12: but as this operation comes under variety 3d, we will defer proof till we treat of that variety. It may be proven by variety 2d, by finding the rate per cent. From the foregoing we conclude, that,

*To find how an article must be sold to gain or*

lose a given per cent.; place the cost price on the right, for the demand: 100 on the left, for the same name; and 100, increased by the gain per cent. added, or diminished by the loss per cent. subtracted, last on the right, for the term of answer.

Purchased sugar at 5 cents per lb.: how must it be sold to gain  $12\frac{1}{2}$  per cent.? Here, 5 cents is the demand; 100 the same name; and  $112\frac{1}{2}$ , the name of answer. The latter is reduced, making  $2\frac{1}{2}$ , and is stated as above.

$$\begin{array}{r} 100 \overline{) 5} \\ \cdot 2 \overline{) 225} \\ \hline 15 \frac{1}{2} \end{array}$$

Again: how must cloth that cost  $\$7\frac{1}{2}$  per yard, be sold to lose 40 per cent.? that is, what will  $\$1\frac{1}{2}$  be reduced to, if \$100 are reduced to 60?

$$\begin{array}{r} 2 \overline{) 15} \\ 100 \overline{) 60} \\ \hline \$14\frac{1}{2} \end{array}$$

If a horse cost \$50, how must he be sold to gain 50 per cent.? Here, the demand is \$50, 100 the same name, and 150, the term of answer.

$$\begin{array}{r} 100 \overline{) 50} \\ 150 \overline{) 150} \\ \hline 75 \end{array}$$

We find that 50 per cent. advances to \$75: or, that it gains \$25.

Let us see if selling another horse that cost \$75, at 50 per cent. loss, will bring 75 back to 50. It is perceived here, as in a former question, that 50 per cent. loss on \$75, brings 75 down to \$37 $\frac{1}{2}$ ; for while 50, at 50 per cent., gains or loses 25, 75, at 50 per cent., gains or loses 37 $\frac{1}{2}$ . This difference, and the reason for it, are too palpable to need further comment.

$$\begin{array}{r} 100 \overline{) 75} \\ 50 \overline{) 150} \\ \hline 37\frac{1}{2} \end{array}$$

If 8 yards of cloth cost \$20, for how much must 200 yards be sold, to gain 20 per cent.? Here, it is manifest that the statement is made, first by proportion, thus,

$$\begin{array}{r|l}
 8 & 200 \\
 & 20 \\
 \hline
 100 & 500 \\
 & 120 \\
 \hline
 & \$600
 \end{array}$$

This being the cost of 200 yards, we go on and say, what will these \$500 be advanced to, if 100, or per cent., be advanced to 120? The answer is 600. It is wholly unnecessary to make two separate statements in this case: for, knowing that the answer is involved in the first statement, that is, that the number of dollars which the 200 yards would cost, could be found by working the question, we say, what will this supposed answer or sum, be yet *farther* advanced to, as the demand, if 100 opposite, be made 120? thus,

$$\begin{array}{r|l}
 2- & \$200 \\
 100 & 20 \\
 & 120-6 \\
 \hline
 & \$600
 \end{array}$$

The \$100, or per cent., placed on the left, is placed opposite the \$20, which term represents the answer, and which is the demand.

$$\begin{array}{r|l}
 6 & 1 \\
 18 & 15 \\
 1 & 3 \\
 8 & 9 \\
 100 & 1200 \\
 2 & 275 \\
 \hline
 & \$775\frac{7}{8}
 \end{array}$$

If  $\frac{1}{3}$  of  $\frac{2}{3}$  of a farm cost \$1200, for how much must  $\frac{1}{3}$  of  $\frac{1}{3}\frac{2}{3}$  of the same be sold, to gain  $37\frac{1}{2}$  per cent.? The statement of the question made, by proportion, nothing more is necessary than to place 100 opposite dollars, or involved answer, and  $137\frac{1}{2}$ , or  $275\frac{1}{2}$ , on the right.

I have 300 lbs. bacon, which cost 5 cents per pound: how must I sell the whole, to gain 25 per cent.? It is necessary in the first place, to find the cost of the whole bacon; consequently, we say, if 1 lb. cost 5 cents, what will 300 lbs. cost? thus,

Then we say, what will all the cts. paying for the bacon, be advanced to, if 100 be advanced to 125?

$$\begin{array}{r|l} 1 & 300 \\ 100 & 5 \\ \hline & 125 \\ \hline & \$18,75 \end{array}$$

Bought 200 galls. oil, at 60 cents per gallon; I lost 40 galls. and wish to know how I must sell the remainder to gain 30 per cent. on the whole investment. What will 200 galls. cost if 1 gallon cost 60 cents? Then we know that after deducting 40 gallons loss, we have but 160, which this sum of money, on the right, has paid for. Supposing this on the right to be the price of the 160 galls., we say, what will one gallon cost on the right, if 160 opposite, cost these cents: the result would be the advanced price at which 1 gallon of the 160 would be sold; so that the whole sale would bring the money first invested in the 200. We then say, what will this price be yet farther advanced to, if 100, or per cent., opposite, be 130? We get the answer in the price of 1 gallon, at 30 per cent. profit, thus,

In the case above, all that is necessary, is to suppose that the demand *does* exist on the right; for we know that this demand is but the result of another proportion, preceding, which we could easily ascertain by making the separate statement. The demand, 1 gallon, is merely supposed; for it is wholly unnecessary to assign a place to a unit, which, so far as the work is concerned, is useless.

$$\begin{array}{r|l} 1 & 200 \\ 160 & 60 \\ 100 & 130 \\ \hline & 187\frac{1}{2} \end{array}$$

## SECOND VARIETY.

If I buy a lb. of butter for 10 cents, and sell it for 15 cents, what do I gain per cent.? The demand here is, what will per centum, or 100 gain, if 10 cents opposite, gain 5? The question is not what will 100 be advanced to, if 10 be advanced to 15; but to know how much 100 will gain. Stating accordingly, the answer will be the gain of 100, or the rate per cent. profit. It is supposed, not unfrequently, that the *per cent.* must be calculated on the selling price, which, in the case cited, would present this absurdity: if 15 cents gain nothing, what will 100 cents, or per cent., gain? We know that the gain has been effected by the use of the 10 cents; and if the 10 cents have gained 5 others, certainly in the same ratio; 100, or per cent., which are 10 times as many, will gain 10 times as much as 5, which is 50 per cent. We could find this rate per cent. profit in the following way: if 10 be advanced to 15, what will 100 be advanced to? In this case, the cost price, 10, is added to the gain, 5, to make the advanced price; therefore, the par per centum, 100, will be added to the gain of the 100, to make advanced per cent. Hence, the necessity, if the question be wrought thus, of subtracting 100 from the answer. This mode of finding per cent. is, however, neither direct nor natural. Here, it is seen, that as we would subtract the par, 10, to leave the gain,

so we would subtract par per cent. to leave the gain per cent. This is presented only for its theory, which will be found applicable in variety fourth. Fifty per cent. answer.

$$\begin{array}{r|l} 10 & 100 \\ & 15 \\ \hline & 150 \\ & 100 \\ \hline & 50 \end{array}$$

We now revert to variety first, and knowing that the one lb. of coffee, which cost 10 cents, was sold at 12, to gain 20 per cent., we will see if this is correct. When purchased at 10 and sold at 12, the gain was 2: now, if 10 cents gain 2, what will 100 cents gain? Twenty, which is 20 per cent., is the answer.

$$\begin{array}{r|l} 10 & 100 \\ & 2 \\ \hline & 20 \end{array}$$

Suppose the question above were thus: if 10 cents lose 2, what is the loss per cent.? It is evident that it would be precisely the same operation, and that 20 per cent. would be the result. Again: If I buy raisins at  $7\frac{1}{2}$  cents per lb., and sell them at 10 cents, what do I gain per cent.? The first thing to do in all such cases is, evidently, to find the gain or loss on the cost, and say, if the cost has given this gain, what will per centum give? Now,  $7\frac{1}{2}$  cents gain  $2\frac{1}{2}$ ; the question is stated accordingly; disposing of fractions in the usual manner. The result,  $33\frac{1}{3}$  per cent., is evidently correct, if we consider that  $2\frac{1}{2}$  being  $\frac{1}{2}$  of  $7\frac{1}{2}$ , the answer should likewise be  $\frac{1}{2}$  of 100.

$$\begin{array}{r|l} & 100 \\ 15 & 2 \\ \hline 2 & 5 \\ - & 33\frac{1}{3} \end{array}$$

If a horse is bought for 40 dollars, and sold for 80, what is the gain per cent.? If 40 gain 40, per cent. will gain 100. The result is 100 per centum.

$$\begin{array}{r|l} 40 & 100 \\ 40 & \\ \hline & 100 \end{array}$$

From the foregoing we conclude that, *To find the rate per cent., profit or loss, when an article is purchased at one price and sold at another;*

- *Ascertain the gain or loss on the cost price, by subtraction: make 100 the demand: the cost price, the same name; and the gain or loss, the name of answer, last on the right. The answer will be the gain, or loss, per cent.*

A bill of goods cost 2400 dollars, and was sold for 3000; what was the gain per cent.? The 2400 gained 600; hence, the per cent. is 25.

Bought cloth at  $4\frac{1}{2}$  dollars, and sold it at  $4\frac{1}{4}$ ; how much per cent. was gained or lost? We find that  $4\frac{1}{2}$  dollars lose  $\frac{1}{4}$  of a dollar: hence, we inquire, what will 100 lose? The rate of loss is  $2\frac{1}{2}$  per cent., thus,

Let this be proven by Variety 1st, by asking to what price will  $4\frac{1}{2}$  be reduced, to lose  $2\frac{1}{2}$  per cent.? That is, what will  $\frac{1}{4}$  be reduced to, if 100 be reduced to  $97\frac{7}{8}$ , or  $\frac{155}{16}$ ? Four and five-eighths is evidently the answer; for this was the price at which it was first sold, at a loss.

Ribbon that cost 6 cents per yard, is sold at  $7\frac{1}{2}$  cents, what is the gain per cent.?

Cloth that cost  $18\frac{1}{2}$  cents per yard, is sold at  $12\frac{1}{2}$  cents; what is the loss per cent.? The  $18\frac{1}{2}$  lose 6; therefore, 100 will lose  $33\frac{1}{2}$ , which is the rate per cent. loss.

It must be observed here, that loss, as well as gain, is made on the cost price.

THIRD VARIETY.\*

7.

Sold a yard of cloth for 250 cents, and thereby gained 50 per cent.; what did it cost? It is manifest that 50 per cent. was calculated on the cost price, and then added to it, for the selling price; so that to take 50 per cent. *from* the selling price, which is considerably larger than the cost price, would be taking off a larger sum than the cost price, at 50 per cent., would give. It is, therefore, necessary to find the cost, and then 50 per cent. reckoned on this and added, would make the selling price. The 250 cents are the advanced price, from which we wish to deduct the per centum that has been added. To do this, we must compare it with the advance value of per cent., or with 100, advanced by the rate, added to it. Now the rate 50, when added to 100, makes 150, for the advance or amount of per cent. If this be the advance or amount of per cent., to find the cost or par, we must reduce it to 100. We have, therefore, 150 advance per cent. to compare by ratio with 250, advanced price of cloth, and conclude, that if 150 advance, be reduced to 100, par or cost, 250 advance must be reduced in the same ratio, to find *its* par or cost. Hence, we make 250 the demand, 150 the same name, and 100 the term of answer: thus,

$$\begin{array}{r|l} 150 & 250 \\ & 100 \\ \hline & \$1,66\frac{2}{3} \end{array}$$

In reducing the 150 above, it is not the 150 losing 50 per cent. ; for this would reduce it to 75, instead of 100 : but it is 150 losing the gain, being brought back to the cost or sum, that first gained it. If 100, at 50 per cent., be 150, certainly 100 of it is original value, and 50 gain ; both making the amount 150. So, likewise, with the cloth ;  $166\frac{2}{3}$  is the first value, and  $83\frac{1}{3}$  the gain : both together, making the amount or advanced price, 250. Now, by proportion we find, that if 100 would gain \$50,  $166\frac{2}{3}$  would gain  $83\frac{1}{3}$ . So, it becomes reasonable, that in reducing an article that has been sold at an advanced, to its cost price, to compare advanced price of the article with advanced per cent. We may now prove this by both preceding Varieties.

$$\begin{array}{r|l} 3 & 500 \\ 100 & 150 \\ \hline & \$2,50 \end{array}$$

*First*, what must a yard of cloth which cost  $166\frac{2}{3}$ , be sold for, to gain 50 per cent. ?

$$\begin{array}{r|l} 500 & 100 \\ & 3 \\ 3 & 250 \\ \hline & 150 \end{array}$$

*Second*, If a yard cost  $1,66\frac{2}{3}$ , and sell for 250 cents, what is the gain per cent. ? We know that this ought to be 50. We say then, if  $166\frac{2}{3}$  gain  $83\frac{1}{3}$ , what will 100 gain ?

It is seen here, that when we take 50 per cent. from 250, we reduce it to  $1,66\frac{2}{3}$  ; and that 50 per cent. on this  $166\frac{2}{3}$ , will elevate it to 250 again. Some think it singular that it can be done in this case, and not in the case of the first example, in Variety First. The great reason of this difference, is the difference of

names. In the latter case, we are falling from the amount to the principal: from the advance to the cost, by reducing the amount of per cent., to its par or first value: in the former, the work has no reference to finding the par or cost price; but merely to laying on or taking off per cent. on larger or smaller sums; as 10 and 12 cents: 5 and 20 cents, &c. In the latter case our supposition is the advance per cent.: in the former, it is not advance, but par per cent., or 100 being reduced to some loss price. Advance, par, loss, reduced, &c., become, therefore, important distinctions if we would inquire the prime reason of different operations, which are rather apparently the same.

Suppose I sell a yard of cloth for 480 cents, and thereby lose 20 per cent.: It is evident that I have sold it for a price, 20 per cent. too small. Now, many would think that we might advance the 480 cents, 20 per cent., and have the cost price; but this is a mistake: for the 20 per cent. loss, is so much on the cost price, and could not be calculated on the 480; because *it* is the reduced price; and 20 per cent. on this reduced price will not make as much as on the cost price. We say therefore, what will this reduced price, 480, be advanced to for par, if 80, the reduced value of per cent., be advanced to 100, par? It is seen here, that we have reduced opposite reduced, and last on the right, par or 100 for the term of answer. We find that the cloth cost 600 cents, thus,

$$\begin{array}{r}
 80 \overline{) 480} \\
 \underline{100} \\
 \text{cts. } 600 \\
 100 \overline{) 600} \\
 \underline{80} \\
 \text{cts. } 480 \\
 600 \overline{) 100} \\
 \underline{120} \\
 \hline
 20
 \end{array}$$

It was this 600 that lost the 20 per cent.; not the 480: and if 600 be reduced 20 per cent. by Variety first, the reduced or loss price will be found 480; thus,

This may be proven again, by Variety 2d, thus: If cloth that cost 600, is sold for 480, what is the loss per cent.? We know that it was 20: hence 20 per cent.

From the foregoing, we conclude, that,

*To find the cost price, when an article has been sold at a specified per cent., gain or loss, make the selling price the demand; 100, increased by the gain per cent. added, or diminished by the loss per cent. subtracted, the same name, opposite; and 100, the term of answer, on the right: the answer will be the cost price.*

$$\begin{array}{r}
 225 \overline{) 500} \\
 \underline{2} \\
 100 \\
 \hline
 4,44\frac{4}{5} \\
 9 \overline{) 4000} \\
 100 \overline{) 225} \\
 \underline{2} \\
 5,00 \\
 4000 \overline{) 100} \\
 \underline{9} \\
 9 \overline{) 500} \\
 \hline
 12\frac{1}{2}
 \end{array}$$

If a man sell a yard of cloth for \$5, and thereby gain  $12\frac{1}{2}$  per cent., what did it cost him? What will 500 advanced price, be reduced to, if  $112\frac{1}{2}$  advanced per cent., be reduced to 100?

If I have a yard of cloth that cost  $4,44\frac{4}{5}$ , how must I sell it, to gain  $12\frac{1}{2}$  per cent.? Variety 1st.

If I buy cloth at  $444\frac{4}{5}$ , and sell it at 500 cents, what do I gain per cent.? Variety 2d. It is evident that  $444\frac{4}{5}$  gain  $55\frac{2}{5}$  cents: then, what will 100 gain? Here,  $12\frac{1}{2}$  is the answer.

If, when wheat is sold at 80 cents per bushel, 20 per cent. is lost, what did it cost? What will the reduced price, 80, be advanced to, if 80 reduced per cent. be advanced to 100, par?

$$\begin{array}{r} \cancel{80} \cancel{0} \cancel{0} \\ 100 \\ \hline 100 \end{array}$$

When wheat is purchased at 100 cents, and sold for 80, what is lost per cent.? It is the 100 cents here, that lose 20: hence 20 per cent. answer.

$$\begin{array}{r} 100 \cancel{0} \cancel{0} \\ 20 \\ \hline 20 \end{array}$$

If hemp, sold at \$4½ per cwt., gains 10 per cent., what did it cost?

$$\begin{array}{r} 110 \cancel{4} \cancel{5} \cancel{0} \\ 100 \\ \hline 4,09 \frac{1}{2} \end{array}$$

If hemp cost \$4,09½ cents per cwt., for how much will it be sold to gain 10 per cent.?

$$\begin{array}{r} 11 \cancel{4} \cancel{5} \cancel{0} \cancel{0} \\ 100 \cancel{1} \cancel{1} \cancel{0} \\ \hline 4,50 \end{array}$$

Purchased flour at \$2,40 per barrel, which was 40 per cent. below cost: what was the cost?

$$\begin{array}{r} 60 \cancel{2} \cancel{4} \cancel{0} \\ 100 \\ \hline 4,00 \end{array}$$

How must flour that cost \$4,00 be sold to lose 40 per cent.?

$$\begin{array}{r} 100 \cancel{4} \cancel{0} \cancel{0} \\ 60 \\ \hline 2,40 \end{array}$$

Sold a horse for 120 dollars and thereby lost 20 per cent., whereas I ought have gained 40 per cent.; how much was he sold under his value? It is plain in the first place, that if I lost 20 per cent. in selling him at \$120, he must have cost me more than this: consequently this is the reduced price. It may be said, therefore, what will this reduced price be

advanced to, for cost, if 80, the reduced per cent., be advanced to 100? This will give the cost price of the horse. Now, this cost price must be advanced 40 per cent. Hence, we say, what will this cost price be advanced to, if 100 be advanced to 140? Thus, the two statements are combined,

80	120	15
100	100	
	140	
	210	
	120	
	90	

We find that the horse should have sold, to comply with these conditions, for 210 dollars; from which, subtracting 120, we have a loss of 90 dollars. We might take the two questions separately; or, as they are combined, we

might drop the hundreds for mere convenience, saying 80, reduced price, may be advanced to 140, selling or advanced price.

Suppose, when a horse is sold for \$120, 20 per cent. is gained, whereas a loss of 40 per cent. might be sustained; how much is he sold over his value?

120	120
100	100
	60
	60

We find that the horse might have been sold for \$60, and is consequently sold for sixty too much, in making the price 120.

120	120
	60
	60

Again: In the latter example the 100 is suspended on each side, while the statement is still quite as perspicuous as before.

Operations in this variety of profit and loss, are quite similar to those of discount. In discount the advance made from principal or par, to amount, is based on the losses of a specified time; whereas, in this case, the advance

depends on a usage of common consent, whereby an individual is allowed to make a reasonable per cent. in lieu of the accommodation extended to those around him.

FOURTH VARIETY.

If, in selling a yard of cloth for \$5, I gain 20 per cent., what will be gained if it sell for \$6?

In this case we may say, what per cent. will be gained by selling at \$6, if by selling at \$5 we gain 20 per cent? Thus,

It would gain 24 per cent. Now 5 advancing to 6 gains 1 dollar; what per cent is it? or, what will 100 gain if 5 gain one?

We find that the simple advance from 5 to 6 is equivalent to 20 per cent. Here the general advance is 24, and the advance from 5 to 6, 20 per cent., which, added, make 44 per cent. as the gain in selling at \$6.

We say above first, what will 6 gain, if 5 gain 20: and, again, what will 100 gain, if 5 gain 1? In the first place 5 gains 20 as a mere addition; and in the second, it gains 1 as a specified per cent. Knowing, then, that we must yet further use the 20 to advance it to a larger amount, and likewise the 100 to get the per cent., we combine the two operations, by adding 100 to the 20 for the purpose of getting the per cent.; suspending one of the fives on the left; and finally, subtracting this

$$\begin{array}{r} \$6 \\ 20-4 \\ \hline 24 \end{array}$$

$$\begin{array}{r} 5|100 \\ 1 \\ \hline 20 \end{array}$$

100 thus added above, which leaves the true answer; thus,

$\begin{array}{r} 120-2 \\ 144 \\ 100 \\ \hline 44 \end{array}$	<p>By thus adding the 20 and 100, we work two propositions, and perform addition at the same time. It is very seldom necessary to add numbers on the line; and is done in this instance only to avoid two statements, and to shorten the work.</p>
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When one price is advanced to another, and a gain made, we subtract 100 from the answer, and take the remainder as the per cent.; and when one price is reduced to another, the answer is subtracted from 100, while the remainder is, as before, accounted the true per cent.

To prove this answer correct, we may find the cost of the yard of cloth, knowing that \$5 is 20 per cent. more than the cost; thus,

$\begin{array}{r} 6-120 \mid 5 \\ 100-5 \\ \hline 4\frac{1}{2} \end{array}$	<p>We find by this, that the cost was <math>4\frac{1}{2}</math> dollars. Now, if <math>4\frac{1}{2}</math> dollars, in advancing to 6, gain <math>1\frac{1}{2}</math> dollars, what is the rate per cent. gain?</p>
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$\begin{array}{r} 110-4 \\ 6 \\ \hline 11 \\ 44 \end{array}$	<p>Thus, by variety second, we find that the original cost was <math>4\frac{1}{2}</math>, and after subtracting this from the selling price, by variety second, find also that the rate is 44 per cent., as above.</p>
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In selling a watch for \$10, I lose 20 per cent; what per cent. would be lost in selling it at \$8?

Here the selling price, 8, is the demand; the first reduced price, 10, the same name; and 100, reduced by the loss per cent. subtracted, the same name. It may be observed here, that *if the per cent. is gain, it must be added, and if loss, subtracted*. Taking 64 above, from 100, we have 36 per cent. loss, for the answer.

$$\begin{array}{r} 100 \\ 10 \times 8 \\ \hline 64 \end{array}$$

We know that if \$10 lose 20 per cent., 8 will lose 16; and likewise in descending from 10 to 8 we lose 20 per cent.: now 20 and 16 = 36, as above.

This may be again proven, thus: What will \$10, reduced price, be advanced to, if 36 reduced per centum be advanced to 100?

Thus the cost price of the watch was \$12½; now, in selling for 8, 12½ loses 4½; hence we ask, what will 100 lose?

$$\begin{array}{r} 100 \\ 12 \frac{1}{2} \times 8 \\ \hline 12 \frac{1}{2} \end{array}$$

The loss in reducing to this price is thus found to be 36 per cent., which proves the work correct in every part.

$$\begin{array}{r} 100 \\ 12 \frac{1}{2} \times 8 \\ \hline 36 \end{array}$$

If, in selling a pound of tobacco for 30 cents, I lose 10 per cent., what will be lost if it is sold at 25 cents?

The result is 75 cents, which we subtract from 100, leaving 25 per cent. as the answer.

$$\begin{array}{r} 100 \\ 30 \times 25 \\ \hline 75 \end{array}$$

Now, if 30 was 10 per cent. loss, we can easily see what it cost; thus,

The cost was 33½ cents.

$$\begin{array}{r} 100 \\ 30 \times 10 \\ \hline 33 \frac{1}{2} \end{array}$$

If  $33\frac{1}{3}$  reduced to 25, loses  $8\frac{1}{3}$ , what will 100 lose?

$$\begin{array}{r|l} 100 & 100 \\ 33\frac{1}{3} & \\ \hline 25 & \\ \hline 25 & \end{array}$$

Again the question is proven by finding the per cent. between the cost and selling price.

If, in selling a horse at \$30, I gain 30 per cent., what will I gain per cent. by selling him at \$40?

$$\begin{array}{r|l} 30 & 40 \\ 130 & \\ \hline 173\frac{1}{3} & \\ 100 & \\ \hline 73\frac{1}{3} & \end{array}$$

per cent., be reduced to 100?

$$\begin{array}{r|l} 130 & 30 \\ 100 & \\ \hline 23\frac{1}{3} & \end{array}$$

We subtract 100, and have for answer  $73\frac{1}{3}$  per cent. We now find, by variety third, the cost of the horse, which in selling at \$30 gained 30 per cent.; saying, what will 30 advanced price be reduced to, if 130 advanced price be reduced to 100?

$$\begin{array}{r|l} 100 & \\ 30 & 12 \\ \hline 12 & 220 \\ \hline 73\frac{1}{3} & \end{array}$$

Now,  $23\frac{1}{3}$  being the cost price, we know that it gains  $16\frac{2}{3}$  in advancing to \$40. Therefore, by variety second, we say, if  $23\frac{1}{3}$  gain  $16\frac{2}{3}$ , what is the rate per cent., or what will 100 gain?

Again, the work is proven correct by subsequent statements.

From the foregoing we conclude, that,

*To find the rate per cent., gain or loss, when an article, sold at a specified price, with a specified gain or loss per cent., is advanced or reduced to yet another price: Make the last selling price, the demand; the first selling price the same name; and 100, increased by the gain per cent. added, or reduced by the loss per cent. subtracted, the term of answer. If a per cent. is gained,*

*subtract 100 from the answer; if lost, subtract the answer from 100, and in either case the remainder will be the true rate per cent. Or,*

*Proceed according to directions in variety third, and find the original cost; then find by subtraction the difference between the cost and the selling price; and by variety second, find the rate per cent. gain or loss.*

This variety of profit and loss is of but little practical value, but may serve to awaken close investigation in the mind of the reader.

#### FIFTH VARIETY.

We shall consider under this head questions of a general character, with particular reference to the combination of several separate statements into one general statement, by which a specific answer may be obtained. As all operations, in this or any other department of profit and loss, depend primarily on proportion, it will be necessary to make all of the statements occurring in combination, by the general principles indicated in simple ratio. Such statements of concatenated proportions, might be called conjoined proportion, which in the strictest sense is but making the answer of a *preceding*, the demand of a *subsequent* proportion. This being the case, and all proportion depending in solution on multiplication and division, we conclude, that these several multiplications and divisions may be made all together, and at the same time.

The statement of questions by combination

gives free exercise to all the analytic powers of the student's mind, and tends greatly to the cultivation of correct modes both of thinking and reasoning. It will be found necessary, in all such questions, to commence with the commencing point, and keep up all of the natural relations of the questions until the term of answer is found.

Sent to Cincinnati for 440 gallons of wine, and paid  $87\frac{1}{2}$  cents per gallon: paid 2 per cent. commission to my agents; and in exchanging specie for depreciated Kentucky paper, gained 10 per cent. premium: lost 40 gallons by leakage: how much must the remainder be sold for per gallon, to gain 20 per cent. on the prime cost?

It is evident that we must find what the 440 gallons cost, at the price, and that we must then increase this sum 2 per cent. for commission; for the commission is reckoned on the sum of money paid for the wine. We know that this depreciated paper will be received in payment for the wine and commission, quite as well, if necessary, as gold; and that \$110 of the paper are worth \$100 of the specie. We, therefore, find what sum of specie will pay for this paper, or cost of the wine and commission. This, then, would give the amount of specie to be sent to Cincinnati; but losing 40 gallons, we divide the whole quantity of specie paying for all the wine, by the number of gallons left, 400, and find the advanced price which each gallon of the reduced lot must sell at, to reproduce the amount expended. We then advance this price 20 per cent.,

and find the selling price of the wine, per gallon. It is a settled matter, that the commission must be paid on the sum invested, whether in specie or paper. It is likewise easily seen, that it is the amount of specie sent to Cincinnati that must be divided by the reduced quantity of wine; for the purchaser merely wishes to know the advanced price per gallon that will reinstate him in his expenditure, without reference to the 10 per cent. discount saved; for if he were to divide the price paid in paper, he would reindemnify his loss of 40 gallons, and retain also the 10 per cent.

But, as it is, he wishes only to make 20 per cent. on the whole transaction, by advancing the specie price of each gallon of the reduced quantity 20 per cent. We consequently make the statement by proportion; thus,

What will 440 gallons cost, if 1 gallon cost  $87\frac{1}{2}$ ; what will this sum, or price of all the gallons, be advanced to, if 100 be advanced to 102, for amount of both commission and payment? Now, what will all this sum of money in paper, which pays for the wine, be reduced to, for specie, if 110 paper, opposite, be worth 100 of specie, on the right? Then, what will 1 gallon cost, if 200, the remainder after the loss, be worth this last sum in specie? Here, the demand 1, is understood, not expressed. Again: What will this price per gallon on the right, as demand, be advanced to, if 100 opposite, be made 120? We find

1	440
2	175
100	
110	102
400	100
100	120
	<hr/>
	1,07 $\frac{1}{2}$

that the wine must be sold at 1 dollar and  $7\frac{1}{8}$  cents per gallon.

The several successive steps, or proportions in this statement, may be made separately, as follows :

		1440 gallons.
		2175 cents price.
Per ct.	100	385,00 whole price of wine.
		102 commission.
Dis.	110	392,70 amount with commission.
		100 specie.
		357,00 reduced amount of specie.
Red. qu.	400	1 1 gallon demand.
		357,00 whole price in specie.
Per. ct.	100	89 $\frac{1}{8}$ price per gallon, red. qu.
		120 20 per cent. profit.
Ans.		1,07 $\frac{1}{8}$ retail price.

The concatenation of the statement is here kept up, except in one instance, when 1 becomes the demand on the right, and the answer of the former question, 35700, is the price of the 400 gallons. We might dispense with this 1; and use it here, only to show the full proportional relations. Hence, we say, if 400 gallons cost \$357,00, what will 1 gallon cost; and get the ratio between the 400 and the 1, and by this ratio, which is  $\frac{1}{400}$ , multiply the price 35700 cents, and thus decrease it to 89 $\frac{1}{8}$  cents. We have shown the absurdity of attempting to multiply together two denominate things: the same reasoning is true with regard to dividing one denominate by another

denominate thing, as *cents* by gallons, or gallons by cents.

Placing the 110 on the left according to discount may seem wrong, until we reflect that 39270 is the amount of paper which pays for the wine; and which we wish to reduce to gold. This amount of paper must be compared with another amount of paper that will equal per centum or 100 in specie; for specie is the par per cent. of exchange; this amount of paper we know is 110; for the par, 100 specie, will pay for this sum of discounted funds. Hence, the operation is one of pure discount.

Bought 5 hogsheads of sugar, containing each 1200 pounds, at  $2\frac{1}{2}$  cents per lb., and lose 1000 lbs.: how must I sell the remainder per lb. to gain  $6\frac{1}{2}$  per cent. on the prime cost?

The separate statements occur in the following order: How many pounds will 5 hhd. make, if 1 hhd. make 1200? what will these pounds come to, if 1 pound cost  $2\frac{1}{2}$  cents? then, what will 1 pound cost, if the 5000 pounds, after the loss of 1000, cost the price indicated by the last answer in cents? what will this advanced price be yet further advanced to, if 100 be advanced to 106 $\frac{1}{2}$ ; thus,

We say 5 times 5 on the right make 25, which goes into 100 on the left 4 times, etc., etc. In this question I again occurs as demand, while 5000 is the same name, and the preceding an-

$$\begin{array}{r}
 1\cancel{5} \\
 : 1\cancel{1}200-3 \\
 2\cancel{5} \\
 2-5000 \\
 4-1000 \\
 \hline
 4125-85-17 \\
 \hline
 1651 \\
 \hline
 3\frac{3}{5}
 \end{array}$$

answer in cents, the denomination of the answer; that when the answer is obtained it may be advanced  $6\frac{1}{2}$  per cent further.

Suppose I purchase 90 yards of broadcloth at \$5 per yard, on a credit of 1 year; but for ready payment am allowed a discount of 10 per cent.: after receiving the cloth I lose 10 yards; how must I sell the remainder per yard, to gain 10 per cent. on the prime cost?

What will 90 yards come to, if 1 yard cost \$5? what will this amount, 1 year hence, be reduced to for ready payment, if 110 opposite be reduced to 100? then, if this price pay first for 90 yards, or, after sustaining a loss, for 80 yards, what price will pay for 1 yard? what will this price of 1 yard be advanced to, if 100 be advanced to 110 for selling price? thus,

$\begin{array}{r} 190 \\ 110 \overline{) 5} \\ 80 \overline{) 100} \\ 100 \overline{) 110} \\ \hline 5\frac{1}{2} \end{array}$	<p>The answer <math>5\frac{1}{2}</math> dollars, or \$5,62<math>\frac{1}{2}</math> cents, is correct; because when we deduct from the cost of the 90 yards, 10 per cent discount, and divide the remainder by 80 yards, we have the cost price of 1 of the remaining yards; thus,</p>
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$\begin{array}{r} 100 \\ 110 \overline{) 5} \\ 80 \overline{) 100} \\ \hline 5\frac{1}{2} \end{array}$	<p>Now, the difference between this price and <math>5\frac{1}{2}</math>, for which it sold, is <math>\frac{1}{2}</math>. We may now prove that in selling at <math>5\frac{1}{2}</math>, 10 per cent. is gained, by variety second; thus,</p>
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$\begin{array}{r} 100 \\ 225 \overline{) 44} \\ 352 \overline{) 180} \\ \hline 10 \text{ per ct.} \end{array}$	<p>If <math>5\frac{1}{2}</math>, gain <math>\frac{1}{2}</math>, what will 100 or per cent. gain? This 10 per cent. gain is made on the amount of money invested, as has been shown; consequently, on the first cost.</p>
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Purchased cloth at \$3 per yard, but being damaged, I was allowed a deduction of 20 per cent.; for what must I sell it to gain 20 per cent.?

The first statement evidently is, what will \$5 be reduced to, if 100 be reduced to 80: the second, what will this price be advanced to, if 100 be advanced to 120; thus,

To some minds the old difficulty is here again presented, of finding the reduced price at 20 per cent. loss, which at 20 per cent. gain, will not produce the

$$\begin{array}{r} 100\cancel{0}5 \\ 2-100\cancel{0}12\cancel{0} \\ \hline \$44 \end{array}$$

former price. We must recollect that when \$5 lose 20 per cent., they will be reduced to a number which will not at 20 per cent. gain a sufficient amount to advance to \$5 again. Five dollars in losing 20 per cent. are reduced to \$4; but \$4 to gain 20 per cent. would advance to \$4.80 cents only. The defect, 20, arises from the \$4 being too small to gain at 20 per cent. the sum that \$5 would gain. The first statement of the question above, if wrought separately, would give \$4, the reduced price of the cloth; hence, when we increase this answer 20 per cent., we make it \$4 $\frac{1}{2}$ .

I purchase another piece of cloth, on which the sum made was 20 per cent.; in consideration of damages he lets me have it at cost price, or at a discount of 20 per cent.: what should I get for it?

Five dollars is the advanced price; hence, we will, by variety third, compare with it 120, advanced per centum, and reduce it to cost or par price, by the par, 100, on the right. Then,

by another combined statement, we will advance it 20 per cent. for selling price; what will the selling price be? The 20 per cent. advance must be reckoned, like all other profits and losses, on the cost price; now, 20 per cent. discount has been taken off for the purpose of finding cost price; thus, when the cost price is found, we must, by 20 per cent., advance it again to its former advanced price, \$5; thus,

$\begin{array}{r} 120 \overline{) 5} \\ 100 \overline{) 100} \\ 100 \overline{) 120} \\ \hline \$5 \end{array}$	<p>The difference in these two operations is, that in the latter case we discount from the advanced to find the cost; whereas in the former, we reduced the <i>cost</i>, by <i>simple loss</i>, to find the <i>reduced</i> price.</p>
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We will now give a few solutions on general principles.

A purchases 500 pounds of sugar at 6 cents per lb.; how must he sell it per lb. to gain \$20 on the whole lot?

It is manifest that it is necessary to find, first, the cost of the sugar, which is \$30.00. To this we must add 20, making \$50, the whole price that the 500 lbs. must sell for, to gain \$20. Now, what will 1 pound cost, if 500 pounds cost \$50; thus,

$\begin{array}{r} 500 \overline{) 1} \\ 500 \overline{) 500} \\ \hline 10 \text{ cts.} \end{array}$	
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A miller sold a quantity of rye at \$1 per bushel, and gained 20 per cent.; soon after, he sold of the same to the amount of \$37.50, and gained 50 per cent.; how many bushels were there in the last parcel, and at what did he sell it per bushel?

We know that \$1 is the cost price of one bushel, with 20 per cent. profit added: consequently, according to variety third, the cost price of the first quantity is  $83\frac{1}{3}$  cents; thus,

Now, \$37,50 is 50 per cent. more than the cost price of the second lot; so, by the same process, we ascertain that the lot cost \$25; thus,

$$\begin{array}{r|l} 120 & 1 \\ & 100 \\ \hline & \frac{1}{3} = .83\frac{1}{3} \end{array}$$

Now, we say, how many bushels will \$25, the cost of the lot, buy, if  $83\frac{1}{3}$ , cost price, buy 1 bushel; thus,

$$\begin{array}{r|l} 150 & 37,50 \\ & 100 \\ \hline & \$25 \end{array}$$

We find that 30 bushels were sold. Now, if these 30 bushels cost \$37,50, what will 1 bushel cost? thus,

$$\begin{array}{r|l} 250 & 37,50 \\ & 3 \\ \hline & 30 \text{ bu.} \end{array}$$

It is found by the following statement that he sold 30 bushels at \$1,25 cents per bushel; thus,

$$\begin{array}{r|l} 30 & 1 \\ & 37,50 \\ \hline & \$1,25 \text{ cts.} \end{array}$$

We see here beautifully illustrated, the distinction between cost, advanced, and reduced prices. We find the cost of each; compare the cost of 1 with the cost of the lot, and find the number of bushels in each lot. Then, knowing that these bushels cost the price of the lot, we find the price of one bushel. This is an easy and simple process of reasoning; yet the conditions and relations of such questions are seldom understood, unless nice distinctions are made in the *terms*.

A merchant bought a parcel of cloth, at the rate of \$1 for 2 yards, of which he sold a cer-

tain quantity at the rate of \$3 for 5 yards; and then found that he had gained as much as 18 yards cost; how many yards did he sell?

We know that the cloth cost 50 cents per yard, and that he sold it for 60 cents; consequently, he gained 10 cents per yard. Now, he gained as much in selling a quantity of it as 18 yards cost; which is 900 cents. If, therefore, 10 cents is the gain of 1 yard, of how many yards is 900 cents the gain? Nine hundred is the demand; 10 cents the same name, and 1 yard the term of answer; thus,

$$\begin{array}{r} 180 \cancel{0} \cancel{0} \\ 10 \overline{) 1800} \\ \underline{10} \phantom{00} \\ 80 \phantom{0} \\ 80 \phantom{0} \\ \underline{0} \phantom{0} \\ 0 \phantom{0} \end{array}$$

We have now sufficiently illustrated all of the practical operations in profit and loss, to enable the careful and reflective student to perfect his knowledge of the subject by examples and experiments of his own, both in theory and practice, in all of the usual business transactions of life.

A great number of questions in business come under the head of variety fifth, which, judiciously arranged and stated, may be easily wrought; and frequently with one-fourth the number of figures required by former methods, and separate statements.

From the foregoing illustrations, we deduce the following

#### SUMMARY OF DIRECTIONS.

*For COMBINATION OF STATEMENTS in Profit and Loss.*

I. *Place first on the right the quantity of the article and the cost price:*

II. *If it is desired to advance or reduce this entire cost, by commission, premium, transportation, drawback, or other consideration, place 100 on the left, and 100, increased or reduced by the per cent., on the right.*

III. *To effect a discount, place 100, increased by the rate, on the left, and 100 on the right: Or, if profit and loss follow, place 100, increased or reduced by the gain or loss per cent., on the right, in the place of the 100 or par of discount.*

IV. *If a specified portion of the article of merchandise be lost, and it is desired to know how a unit of the quantity must be sold, subtract the quantity lost, and place the remainder on the left.*

*Or, Make the whole statement a concatenation of proportions, and proceed according to the specified directions in the various rules involved.*

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## DISCOUNT.\*

DISCOUNT is reckoned by two methods; one true; the other false. The false method is very often used by business men; which is merely to reckon the interest on the amount,

\* Discount is from the French *décompte* to count back, and is used synonymously with *rebate*, which is from *rebattre*, to strike off. It implies the striking of a portion from an amount made of separate sums, as is the case in discount, where the amount is composed of the original principal, and the interest supposed to have accrued. Amount is from *monter*, to ascend; which is from the root of the Latin, *mons*, a mountain.

and deduct it for the discount; making the remainder the present worth. By the true method, the amount is reduced to such a sum, as, connected with its own interest for the time, and at the rate, will restore the amount. In other words, the interest on the present worth, which is equal to the Discount, will, if added to the present worth, restore the original amount or debt. Some individuals use both of these methods: the former, if they are paying out money; the latter, if receiving it.

The theory of discount is based on the supposition, that the debt becomes due at a future period, and bears interest from date, or from the specified time when it is to be paid. Therefore, it is necessary to use some standard of present and future value, such as 100, or per centum. One hundred is the value or standard at present time; but 100, with its own interest for the *time* and *rate*, added, will be the future value, of such standard. Suppose the time 1 year, and the rate 10 per cent.: the standard of future value will be 110. This 110 is both principal and interest, which added, make the amount which is always the future value. In the same manner, the debt on which the discount is to be made, is both principal and interest, or amount: hence, the propriety of comparing amount of debt with amount of standard, and by proportion, reducing the amount of debt to its par or present value. In the supposition above, if 110 dollars, one year hence, be reduced to 100 dollars, present time and value, what will any other amount, as \$100 debt, be

reduced to, in the same proportion: that is, if \$110 be reduced to \$100, for present worth, what will 100 dollars be reduced to for present worth? Here, 100 is the *demand*; 110 the *same name*; and 100 the *term of answer*, or present worth. We state accordingly, and the answer will be the sum of money payable at present time.

$$\begin{array}{r|l} 110 & 100 \\ & 100 \\ \hline & \$90\frac{1}{11} \end{array}$$

By annexing two ciphers, this answer might be obtained in cents. The present worth is  $90\frac{1}{11}$  dollars; or 90 dollars, 90 cents, and  $\frac{1}{11}$ . This sum with interest for one year, at 10 per cent., will amount to 100 dollars; which proves the position correct, that *the interest on the present worth is equal to the sum of discount*. It is necessary to make a distinction between the terms used. The *sum* is the whole of anything, from *summum, the whole*: as the sum of *interest*, the sum of present worth, the sum of discount, &c. The *amount* is the result of two or more sums added, as present worth and discount added, which make the amount of debt. *Amount* is from the French *monter, to ascend*. The present worth is the portion of the debt remaining, after the discount is deducted.

It is necessary, in stating this sub-division of numbers, to place amount opposite amount, on the line, that we may ascertain the ratio between such different amounts, and apply it to the par, present worth, or 100, and increase or decrease it accordingly.

What will be the present worth of 400 dollars, 10 years hence, at 6 per cent? Here, as

in all other cases of discount, the interest on 1 dollar, for the time, and at the rate, must be ascertained, and added to 100 cents; and the amount placed on the left of the line. One hundred cents, in 10 years, at 6 per cent., amounts to 160 cents; or, 100 dollars, in 10 years, at 6 per cent., will amount to 160 dolls. It may be observed here, that the denomination of the amount on the left, is determined by the demand on the left. If the demand is dollars, the amount is the same; and if cents, the amount is cents; the left, or amount, being merely an indenominate standard.

It is manifest that 400 is the demand; 160 the same name; and 100 the term of answer,

$$\begin{array}{r} 4-160 \overline{) 400} \\ \underline{160} \phantom{00} \\ 240 \\ \underline{240} \\ 0 \end{array}$$

thus,

This 250 dolls., in 10 years, at 6 per cent., will gain 150 dollars, which, added to the

250, restores the original amount, 400.

What is the present worth of 824 dollars, due 8 months hence, at  $4\frac{1}{2}$  per cent.? We ascertain first the interest on 1 dollar, at  $4\frac{1}{2}$

per cent., thus,

$$\begin{array}{r} 1 \\ 2-12 \overline{) 1} \\ \underline{12} \\ 0 \end{array}$$

3 cts.

We now add this 3 cents to 100, making 103, which we place on the left of the line, thus,

$$\begin{array}{r} 103 \overline{) 824} \\ \underline{103} \phantom{00} \\ 721 \\ \underline{721} \\ 0 \end{array}$$

\$800

The answer is 800 dollars, on which, in 8 months, at  $4\frac{1}{2}$  per cent., the interest would be 24 dollars. This added

makes the amount 824. When it is necessary to ascertain the sum of discount, subtract the present worth from the amount due.

What is the present worth of \$20,86,24, due in 18 days, at 6 per cent. discount? We find the amount of 100 cents, thus,

The amount is 100,3. This number is placed on the left of the line, thus,

$$\begin{array}{r} 1 \\ 100 \overline{) 18-3} \\ \underline{\phantom{00}3} \text{ mills.} \end{array}$$

In this instance, we have 3 mills on the left, after the cents, and on the right four

$$\begin{array}{r} 100,3 \overline{) 20,86,24} \\ \underline{\phantom{00}100} \\ \$20,80,0 \end{array}$$

hundredths of cents. Having this one decimal more on the right, than on the left, one figure must be cut off, on the right, for hundredths. This might be obviated, by using another cipher on the left, making 3 mills 30 hundredths. In such case, the demand and same name, would be of the same denomination. From the foregoing considerations, we are justified in making the following

DIRECTIONS FOR DISCOUNT.

*To find the present worth of an amount of money at discount,*

*Ascertain, by interest, the amount of 100 dollars, or cents, for the time, and at the given rate: or ascertain the interest on one dollar for the time and rate, and add it to 100; make the amount of the debt the demand; the amount of 100 the same name; and 100 the term of answer: the answer will be the present worth. To ascertain the Discount, subtract the present worth from the amount. If the number of decimal places in the amount on the right is greater than in the amount on the left, cut off such surplus in the answer. Cancel as in*

*other cases ; or, if this be impracticable, multiply and divide.*

It may be well to offer a few remarks to those who suppose that Discount and Interest are the same ; or who think that deducting the interest, is a fair method of discounting.

The true discount of 100 dollars, for ten years, at 10 per cent., would be 50 dollars. But the discount by the false method, of subtracting the interest, would of itself, be 100 dollars ; leaving nothing for present worth. The absurdity of this may be better seen, by taking the discount on the same sum for 20 years, at 10 per cent. ; in which case, the discount would be 200 dollars : so that, if deducted, would leave the *holder* of the note 100 dollars in debt to his *creditor*, by receiving payment, or making settlement : whereas, by the correct method, the discount *never can* entirely consume the debt ; as there must *always* be a *present* value. We have no space for descanting farther on the beautiful theory of this sub-division of Arithmetic ; and will proceed to the consideration of *per cent.*, under other heads.

In BANK DISCOUNT, the interest is reckoned on the face of the note, and deducted : the remainder is the present worth, or sum drawn. This is allowing a greater rate per cent. than is specified in the note ; and on what plea, in morals and justice, I am unable to learn.

FACE OF NOTES GIVEN IN BANK.

It frequently becomes necessary to find the face of a note given in bank, to draw a specific sum of money. Suppose the rate per cent. discount be 6; then \$100 face of note will give 94 dollars ready money. Suppose it is desired to draw 4700 dollars. We say, therefore, what will 4700 dollars ready money be advanced to for face of note, if 94 dollars ready money be advanced to 100 dollars, face of note? thus,

Now, the interest at 6 per cent., for 1 year, deducted from this 5000 dollars, will leave 4700 dollars, the sum to be drawn.

Suppose the rate to be 4 per cent., and it is desired to draw 1800 dollars: we know that 96 dollars will be the sum drawn for the face 100, and state accordingly; saying, what will 1800 be advanced to, if 96 be advanced to 100? thus,

Here, the factor 12 into 96 eight times, and into 180, fifteen

times; again, 4 into 8 twice, and into 100, 25 times; while 2 on the left into 10 on the right, five times; which, multiplied thus,  $5 \times 15 \times 25$ , makes 1875, the answer. The interest on this sum, at 4 per cent., is 75 dollars, which deducted, leaves 1800, the face of the note. Hence, *To find the face of a note given at bank, to draw a specific sum,*

*Deduct the interest of 100, for the time and at*

$$\begin{array}{r} \cancel{5000} - 100 \\ \hline \$4900 \end{array}$$

$$\begin{array}{r} 1800 - 15 - 5 \\ \hline 1875 \end{array}$$

*the rate, from 100; place the remainder on the left for the supposition, cash drawn; the sum to be drawn in cash, on the right, for the demand; and 100, face of note, last on the right, for the term of answer: the answer will be the face of the note.*

Bankers generally ascertain the face of the note, (though they frequently wish to avoid it altogether) by a species of approximation, by casting interest on the sum, and subsequently on each separate sum of interest, till the result is too small to be noticed further. The precise face of the note could never be obtained in this way.

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### COMMISSION, BROKERAGE, &c.

Commission, Insurance, Brokerage, Taxes, &c., are wrought by Proportion, and in their standards of value, are based on *per centum*. Transactions of this kind are generally made without reference to time, that is, the time required for the specified transaction, is considered a unit.

Commission is a specified sum paid per 100, for the purchase and sale of merchandize, &c. The rate of commission varies from 1 to 20 per cent. The sum paid for commission is called *bonus*, which is the amount of reward for the trouble incurred; from the Latin, *bonus*, good.

A. sends to B. 500 dollars worth of books,

to be sold on commission, and agrees to allow him  $2\frac{1}{2}$  per cent. commission, what sum does B. receive? Five hundred, the sum, is the demand; 100, or per cent., the same name; and  $2\frac{1}{2}$ , the rate bonus, the term of answer. The answer is consequently  $12\frac{1}{2}$  dollars.

$$\begin{array}{r} 100 \overline{) 500} \\ 2 \overline{) 5} \\ \hline \$12\frac{1}{2} \end{array}$$

The same name and the term of answer, are conformed in their denomination, to that of the demand: for being only standards, they have no specific names, and may become dollars or cents, as indicated by the name of the demand.

What is the commission on 750 dollars worth of wheat at  $3\frac{1}{2}$  per ct.?

Here, we suspend the 100 on the left, and cut off 2 figures for decimals of a dollar in the answer.

$$\begin{array}{r} 750 \\ 4 \overline{) 15} \\ \hline 112,50 \\ \$28,02\frac{1}{2} \end{array}$$

What is the commission on 800 bushels of wheat, worth 60 cents per bushel, at  $6\frac{1}{2}$  per cent.?

In this instance, we place the number of bushels, and the price, on the right, which is equivalent to multiplying

$$\begin{array}{r} 100 \overline{) 500} - 2 \\ 60 \\ 4 \overline{) 25} \\ \hline \$30,00 \end{array}$$

them; and which makes the statement the following: what will all the cents that 800 bushels cost, pay for commission, if 100 cents pay  $6\frac{1}{2}$  cents commission? The 100 might again be suspended on the left, and two more figures cut off in the answer, for hundredths of cents.

A factor receives 708 dollars and 75 cents,

and is required to purchase iron at 45 dollars per ton; he is to receive 5 per cent. commission on the money paid: how much iron will he purchase?

The demand is the amount of money, 708,75 cents, and the same name, 100 with the commission added, or 105; and the name of answer, 100. This would give the amount to be invested. In this instance it would be improper to charge commission on the whole sum of money, that is, to charge commission on the commission received. The question is, what sum must the factor invest in iron, so that the commission on the same, would make such sum amount to 708,75, the original amount of capital. We know that for every 100 dollars that he invests, he receives \$5 commission: this added makes 105: now, this \$105 capital will make 100 investment; and we say accordingly, what must 708,75 capital be reduced to for investment, if 105 capital, be made 100 investment money? thus,

$$\begin{array}{r|l} 105 & 708,75 \\ 45,00 & 100 \\ \hline & 1 \end{array}$$

15 tons

This will give the whole number of cents that may be invested: hence, we say again, how many tons iron will all these cents, in this involved or implied answer, buy, if 4500 cents opposite, buy 1 ton? Here, we combine the two statements in one, and have for the answer, 15 tons. That the commission should be charged on the sum invested only, may be better illustrated by the following contrast: A. sends to B. \$100 worth of books to be sold on commission at 25 per cent.: what commission does B. receive? It

is manifest, that as B. has the trouble of selling the whole lot of books, that part which pays his commission as well as the other, he should receive commission on the entire 100 dollars worth. The commission is consequently, \$25. Again :

Suppose A. send to B. \$100 with which to purchase books : what is the commission at 25 per cent. ? Here, the commission being ready at hand, the agent has no further trouble than to deduct his commission, and invest the remainder. It would be manifestly unjust for him to charge commission on his commission, with which he had invested no time. We will suppose that if he wished to purchase \$100 worth of books, he would necessarily send to B. 100 to invest, and 25 to pay commission, or 125 capital to make 100 investment. If then, \$125 capital make \$100 investment, what investment will \$100 capital, make ?

It is thus ascertained that B. must invest \$80 and reckon his commission on the same ; which, at 25 per cent., would be \$20 ; consuming the entire \$100.

$$\begin{array}{r}
 \$-125 \quad 100-4 \\
 \hline
 100-20 \\
 \hline
 \$80
 \end{array}$$

The difference consists in the medium that the factor operated on ; the one, ready capital ; the other, merchandize, which must first be converted.

## BROKERAGE.

Brokerage is but another form of commission, in which the factor or agent operates in monies, stocks, &c. The rates of brokerage vary from  $\frac{1}{4}$  to 10 per cent.

Brokers operate in two ways: by keeping a current account with their dealers, in which they charge the various sums of premium due them; or by deducting the premium from the capital before investment, as in the case of the purchase of books just mentioned.

What will be the premium or bonus for purchasing 300 shares Whitewater canal stock, worth \$50 per share, at  $1\frac{1}{2}$  per cent. premium?

Here, the number of shares is multiplied by the price per share, 100 placed opposite, and the rate,  $1\frac{1}{2}$ , or  $\frac{3}{2}$  last on the right. We might again suspend the 100 on the left, and cut off two decimals for it on the right. Either method may be used with ease and safety.

B. has 200 shares Illinois Canal stock, which he wishes to sell 20 per cent. below par, and agrees to pay to his broker  $\frac{7}{10}$  per cent., for effecting the sale: stock worth \$100 per share.

The number of shares is here multiplied by the price, and the whole sum of stock reduced 20 per cent., as in profit and loss, which gives the true value of the 200 shares. Then we say, what premium will all of these dollars give, if 100 opposite give  $\frac{7}{10}$  of a dollar? Hence the answer 112 dollars. We might suspend the 100, and the  $\frac{7}{10}$ , and ascertain the discounted value of the 200 shares stock, only.

If my broker purchase for me 300 shares Railroad stock at 10 per cent. advance, and

$$\begin{array}{r}
 300 \\
 100 \overline{) 300} \\
 \underline{200} \\
 100
 \end{array}$$

\$1250

$$\begin{array}{r}
 200 \\
 100 \overline{) 200} \\
 \underline{100} \\
 100 \\
 \underline{80} \\
 20
 \end{array}$$

\$112

charge me 1 per cent., brokerage on the sum invested, what will my stock cost me?

Here, the advance is added to the 100 on the right; whereas, in the question above, the 20 per cent. was subtracted from the same number. The one per cent. added to the 100 on the right, is not to ascertain the premium, but the whole price to which the stock is advanced; and is identical with Variety 1st, in Profit and Loss.

	300
100	100
100	110
	101
<hr/>	
	\$33,330

From the foregoing considerations, we are justified in making the following

SUMMARY OF DIRECTIONS,

*for calculating bonus and premium on Commission, Brokerage, Stocks, &c.*

*To ascertain bonus, premium, &c., Place the amount of money, merchandise, stock, &c. on the right, for the demand: 100 on the left, for the same name; and the rate, bonus or premium, last on the right, for the term of answer; the answer will be in the denomination of the amount: or, suspend the 100 on the left, and cut off two figures in the result, for hundredths. Again:*

*To find the sum of money to be invested, after the commission is deducted from a given amount, Proceed as in discount: and make the amount the demand: 100 with the per cent. commission or brokerage added, the same name; and 100 the term of answer. The answer will be the sum to be invested.*

*To ascertain premium, or selling price of stock, when effected by gain or loss on par,*

*Place the number of shares, and the price per share, on the right: 100 opposite; and 100 increased by the gain per cent., or, reduced by the loss per cent., on the right: then, to ascertain the premium, place 100 on the left, and the rate on the right; or, to ascertain the selling price of the stock, place the 100 on the left, as before, and 100 on the right, increased by the gain, or reduced by the loss per cent. The answer will be the price of the stock, including gain or loss, and brokerage.*

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## INSURANCE.

Insurance is security against hazard or loss of property on land or sea, and is usually divided into two kinds: Fire and Marine.

Fire Insurance is that which secures buildings and other property on land; Marine, which is from *maris, the sea*, is to secure vessels, boats, cargoes, &c., on sea, or on rivers, lakes, &c. The propriety of insurance is found in the fact, that by a single accident an individual may lose his entire property, with little or no hope of recovery: whereas, if he pay a small bonus to an association, his losses may be restored to him, by the association paying him from a large common stock fund; which fund is kept up by the bonus paid by each individual who insures. Reason dictates that it is better to pay a small share of our profits for certain safety, than saving it, to be continually subject to lose, not only the small sum so saved, but the entire capital on which the hope of all gain is based.

Property is insured in two ways : by corporations, which are legalized associations of individuals, with specific powers and privileges, based on a definite amount of common stock fund : and by individuals, according to private contract. Insurance effected in the latter way, is called, "out-door" insurance ; and is never so safe as the former, except when the insurer is careful not to insure property to a larger amount than his personal property would indemnify, in case of loss, on part of the assured.

The instrument of writing given to an individual as evidence of his insurance, is called a Policy : and the sum paid by such individual for insurance, is called Premium. The latter is always a certain per cent. on the value of the property insured. The agent of the company, or the individual who signs this policy or contract, is called an *underwriter*.

The question arises, what per cent. should be paid for the insurance of property ? This is always according to circumstances ; as property is more or less subject to damage. Hence, insurers divide property into *hazardous*, *not hazardous*, and *extra hazardous* ; charging different rates, according to the degree of exposure. These distinctions refer more particularly to fire insurance on land ; as, in marine insurance, there is but little difference in degree of hazard.

1st: A stone or brick building situated remote from other buildings, is, from its location, and the quality of material of which constructed, less liable to destruction by fire, than similar buildings otherwise located ; or buildings

of a different material in the same place. For such, the rate of insurance would be low.

2d : Buildings in blocks, densely surrounded, where fire may communicate from one to another, and constructed of more destructible materials, being more hazardous, are insured at higher rates.

3d : Buildings devoted to purposes involving greater danger, such as chemical manufactories, drug establishments, foundries, &c., being greatly exposed, pay yet higher, and extra rates. In some cases of similar nature, it is impossible to effect insurance at any rate.

The unit of time for insurance on property, is *one year*. Vessels and their cargoes are usually insured for the voyage. Coasting vessels are generally insured by the year; being less liable to loss than out-sea vessels. Coast-ers are insured at rates varying from 3 to 8 per cent.; the difference being the result of a state of peace or war in the vicinity. Whale ships are insured by the voyage, at from 4 to 10 per cent. Insurance on goods, ships, stores, manufactories, chattels, dwellings, &c., varies from  $\frac{1}{2}$  to  $2\frac{1}{2}$  per cent., per annum, according to exposure.

The amount of insurance taken on property, is always less than the property is worth; and is generally not above  $\frac{1}{2}$  of its assessed value. This leaves still a partial degree of risk on the owner, as well as on the insurance company, which keeps him awake to the safety of his property. But were the whole value insured, or an amount greater than this, it would be an inducement to some, to destroy their property,

for the base purpose of converting it into money, or of realizing a small profit on the capital invested in it. Thus, it becomes necessary to insure a definite amount, which *entire* amount may be recovered, if it is shown that such an amount of property has been lost. But if less than this amount is lost, the whole insurance cannot be recovered; only a share proportional to the damage sustained. For instance; if I insure property to the amount of \$1000, and lose one half of it, I can recover but \$500 insurance.

If by fire or other accident, a loss occurs to property, and does not exceed 5 per cent., it is sustained by the owner; otherwise, he might carelessly consign everything to wreck around him, knowing that the company would have to pay for repairs.

It is supposed by some who have little experience, and less common observation, that if insurance companies can make money by taking risks, *individuals* certainly can by *running* risks; but nothing is more illogical. A company with a large common fund, may locate their risks in a great number of places, so that a loss in one place will be more than overbalanced by the gains in another; for it is scarcely reasonable to suppose that losses will occur simultaneously in a great number of places, and on the particular property insured by a certain company. But suppose an individual lose all his property in one place: he is not likely to have enough in another to retrieve this loss by equal gains. And suppose he lose *all*, which not unfrequently occurs, he

is prostrate; his energies tied, whatever they be; for his gain-producing property or element is gone. Now, as before said, which is better, that he save a little and risk the loss of all, or lose a little, and secure the safety of the balance? Common sense would pronounce the former insanity, in all cases of considerable hazard.

Let the economical merchant in Upper Missouri invest his whole worth in a stock of goods in St. Louis, and ship them to Lexington without insurance. The boat is old and pretty well insured, and the owner would be willing to lose her, if by her sinking he would be enabled to purchase a new one; he sinks her: or, a good boat may be shattered on a destructive stump, or burned: his goods are lost; his money gone; and too, his friends, one by one, or *en masse*, have disappeared; what resource is left him of the earnings of former years of toil, but the peaceful shades of undisturbed poverty! He was too eager to be rich; whereas, had he insured, he might, on the recovery of his money, have reinstated himself in business, at the defiance even of accident.

Insurance, like other divisions of Arithmetic involving per cent., is wrought by Simple Proportion. One hundred, or per centum, is the given sum that gives a specified premium. Any other sum, therefore, will, as it is larger or smaller than 100, gain a greater or less sum of premium. For instance:.

## VARIETY FIRST.

A building is insured at the valuation, 2000 dollars, at  $1\frac{1}{4}$  per cent. premium. Now, it is quite evident that the  $1\frac{1}{4}$  premium is gained by 100, or it could not be so much *per centum*. The question may, therefore, be stated in proportion, thus: What premium will insure 2000 dollars worth of property, if  $1\frac{1}{4}$  dollars insure 100 dollars worth of property? The demand is 2000, the same name 100, and the term of answer  $1\frac{1}{4}$  premium. The answer will be the premium for insurance in dollars. The premium is \$25.

$$\begin{array}{r|l} 100 & 2000-5 \\ \hline 100 & 15 \\ \hline & \$25 \end{array}$$

Again: what is the sum premium for insuring a coasting vessel worth 3000 dolls., at  $7\frac{1}{2}$  per cent.?

$$\begin{array}{r|l} 100 & 3000-5 \\ \hline 2 & 15 \\ \hline & \$225 \end{array}$$

What premium must be paid on a shipment of goods from New Orleans to Havre, worth 6,280, at  $2\frac{1}{4}$  per cent.?

$$\begin{array}{r|l} 2-100 & 6280 \\ \hline & 25 \\ \hline & \$157 \end{array}$$

What is the annual insurance on a cotton factory worth 80,000 dollars, at  $\frac{3}{4}$  per cent.?

It is perceived here, that the rate is placed last on the right; consequently, the answer is in premium. If the amount insured is dollars, the answer is dollars; if cents, the answer is cents; because if the demand is dollars or cents, we place 100 dollars or cents opposite, and the premium must be a given sum on 100, of the same

$$\begin{array}{r|l} 100 & 80,000-2 \\ \hline 4 & 3 \\ \hline & \$600 \end{array}$$

denomination. We might obtain the same result in the cases above, by suspending the 100 on the left, and cutting off two figures at the right of the result for decimals of dollars or cents, as the case might be. Thus, in the last example :

$$\begin{array}{r} 80,000-2 \\ 43 \overline{) } \\ \hline \$600,00 \end{array}$$

Here, we cut off the two decimals of a dollar, which are found on the right as the consequence of suspending the 100 on the left. But when the rate is fractional, which is not unfrequently the case, we must, to multiply by it with facility, place the denominator on the left of the line. In cases of prime numbers, or of odd cents in the sum, it may be well to drop the 100, and place on the right, the sum and rate only. What is the premium on a whaler for the voyage, worth 9783 dollars?

$$\begin{array}{r} 9783 \\ 7 \overline{) } \\ \hline \$684,81 \end{array}$$

The 100 could not be easily divided by in this case; hence, it is dropped, and two figures are stricken off for it in the answer. In fact, it may be said to be used for the purpose of keeping up the proportional statement, more than anything else.

A. wishes to purchase a coasting vessel worth 8000 dollars, and insure it for one year at  $7\frac{1}{2}$  per cent.: what sum of money will be necessary to pay both purchase and insurance? This may be done in two ways: first, by finding the premium, and adding it to the cost of the vessel; 2d, by saying, if 100, cost price, amount to  $107\frac{1}{2}$  cost and premium, what will 8000 cost, amount to, with premium?

We find that the cost of this vessel, with the premium added, amounts to 8620 dollars: hence we infer that the premium is 620 dollars.

$$\begin{array}{r} 1000000-2 \\ 431 \\ \hline 8620 \end{array}$$

Questions such as the following frequently occur, when it is quite convenient and simple to combine statements. Purchased 18000 lbs. of cheese at  $5\frac{1}{2}$  cents per lb., on a credit of 1 year; but for ready money I am allowed 10 per cent. discount. I ship the cheese to Nashville, and pay  $1\frac{1}{2}$  per cent. insurance: what will the cheese cost me in Nashville, exclusive of freights?

In this statement, we find the number of cents that the whole quantity of cheese comes to, by multiplying by  $5\frac{1}{2}$  or  $\frac{11}{2}$  cents: I then dis-

$$\begin{array}{r} 18000-3 \\ 110 \\ 110 \\ \hline 300 \\ \hline 915,00 \end{array}$$

count 10 per cent., by saying, what will all these cents be reduced to for present worth, if 110 be reduced to 100? Having now the present cost of the cheese, we say, what will all these cents be advanced to, if 100 be advanced to 101 $\frac{1}{2}$  for insurance, and find that my answer is 915,00 dollars. The two figures are cut off for cents, because the price was in cents, and all the operations have been performed on cents. Were the freight in this case 10, 15, or any other rate per cent., it might be added to the 101 $\frac{1}{2}$ , making 111 $\frac{1}{2}$ , 116 $\frac{1}{2}$ , &c., which would be placed on the line as above.

## VARIETY SECOND,

In insurance, teaches the method of finding the rate per cent. at which an insurance is effected, when the premium and sum are given; as in the following: Paid 60 dollars premium for the insurance of a building, valued at 3000 dollars: what was the rate per cent.? The question here is, what will be the premium on 100 dollars, or per cent., if on \$3000 it is 60 dollars? Hence, per cent., or 100, is the demand, 3000 the same name, and 60 dollars premium, the term of answer. We wish the answer in premium, thus,

$$\begin{array}{r|l} 3000 & 100 \\ \hline & 60 - 2 \\ \hline & 2 \text{ per ct.} \end{array}$$

$$\begin{array}{r|l} 100 & 3000 \\ \hline & 2 \\ \hline & \$60 \end{array}$$

It is found by this, that the rate insured at is 2 per cent., which may be proven as follows: if 100 gains 2 premium, what will 3000 gain?

The answer is here, 60 dollars, the sum of premium first ascertained. This ex-

ample, proving the other, is wrought according to Variety 1st, in Insurance. From the nature and statement of the former question, it may be inferred that,

*To find the rate per cent. of Insurance, when the value of the property and premium are gain, Make 100 the demand: the sum insured, the same name; and the premium, the term of answer: the answer will be the rate per cent. premium.*

If a man pay 40 dollars per annum for the insurance of his house, worth 1200 dollars, what rate per cent. does it cost him?

The rate is  $3\frac{1}{4}$  per cent.; for this is the price of insurance for 100 dolls. worth of property. This question could be proven in the same manner as the one above.

$$\begin{array}{r|l} 3-1200 & 100 \\ & 40-3\frac{1}{4} \\ \hline & 3\frac{1}{4} \end{array}$$

## VARIETY THIRD,

Is to find the sum insured, when the premium and rate are given. Thus, if I pay 90 dollars premium at 3 per cent., what is the sum insured? It is clear, in this case, that 3 dolls. premium, require 100 dolls. worth of property; now, therefore, what will 90 dollars premium require? We consequently make the whole premium, the 90 dollars, the demand: the rate of premium, or per cent., the same name; and \$100, value of property insured by the 3 per cent., the term of answer. The answer is found, therefore, in amount of property, thus,

We find by this, that the amount insured is \$3000; which may be proven by showing that the premium on this sum, at 3 per cent., is 90 dollars. Therefore,

$$\begin{array}{r|l} 3 & 90-3 \\ & 100 \\ \hline & \$3000 \end{array}$$

*To find the amount of property insured, when the rate and premium are given, make the whole sum paid for the premium, the demand: the rate per hundred, the same name; and 100 the term of answer. The answer will be the value of the property insured.*

An importer paid \$700 premium, on wines imported from Madeira to Cincinnati, which

was  $1\frac{1}{4}$  per cent. on the sum insured; how much did he insure?

$\begin{array}{r} 700-2 \\ \$4 \\ \hline 100 \\ \$56000 \end{array}$	<p>We find that the cargo of wines was worth \$56000. This may be proven by finding the premium on this sum at <math>1\frac{1}{4}</math> per cent., by Variety 1st, which would be \$700: or by ascertaining the rate of premium by Variety 2d.</p>
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#### VARIETY FOURTH,

Is somewhat different. By it we ascertain what sum must be insured on a specified amount of property, to recover both property and premium in case of loss.

A. owns a coasting vessel worth \$8400, and wishes to insure at 4 per cent., so that if lost, he may recover both the value of the vessel and the premium: what sum must he insure on? While the rate of premium is 4 per cent., nothing is plainer than that on a policy of \$100, only \$96 worth of property can be insured. Therefore, if \$96 worth of property, be advanced to \$100, property and premium, the question arises, what will \$8400 worth of property be advanced to for both property and premium? 8400 is the demand, 96 the same name, and 100 the term of answer, thus,

$\begin{array}{r} 968400 \\ 100 \\ \hline \$8750 \end{array}$	<p>One hundred here, is the amount, properly speaking, of the property and premium; therefore, the answer will be such an amount as must be insured on, to secure both. Hence the result, \$8750. To prove this correct, we will find that the premium on this sum at 4 per cent., is</p>
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\$350, which subtracted, leaves the first value of the vessel. Therefore, we conclude, that,

*To find a sum to be insured that will secure both property and premium, in case of loss, make the value of the property the demand: one hundred, reduced by the rate per cent., the same name; and 100 the term of answer. The answer will be the value of the property after the premium has been reckoned and deducted.*

Suppose I send to California an adventure of \$19000 worth of goods, at an insurance of 5 per cent.: what sum must be insured on, that in case of total wreck, I may gain both the original value of the goods and the premium?

Here, as in the case above, 95 is the sum of property which requires for amount of property and premium 100: hence, we say, what amount of both will \$19000 property require? Thus, it is seen that 20000 dollars worth must be insured. It may be proven by finding that the premium on 20,000 at 5 per cent., is \$1000. This variety of operation is precisely identical with that of finding the face of a note given at bank, to draw a specific sum. A similar process is used to find the amount to be collected by tax-gatherers, when it is desired to pay both the taxes and the commission on them.

*Life Insurance* is reckoned as other insurance; being a stipulated per cent., according to the age and health of the assured.

*To find the amount to be collected on taxes, to pay both tax and commission, proceed as in Variety 4th in commission, as above.*

$$\begin{array}{r|l}
 \$-95 & \$19000 \\
 \hline
 & 100-5 \\
 \hline
 & \$20,000
 \end{array}$$

## TOLLS.

Tolls are charges made for the transportation of merchandise, &c., by canals, and railroads: and are generally a specific sum per mile, on the 1000 lbs., or the number of bbls., number of perches, number of feet, &c., &c. Whatever becomes the standard of a given article, as 1 barrel, 1 cord, 100 feet, 1 perch, 1000 pounds, &c., must be placed on the left of the line in working the question. For instance; what will be the tolls on 14000 lbs. of flour, 20 miles, at 14 mills per mile on every 1000 lbs.?

20	
14000	14000
14	14
\$3,92,0	

Here, we place down the number of miles first on the right, and multiply it by the number of lbs., by placing the latter under the former. The 20 times 14000 would

be the whole number of lbs. to be carried 1 mile: now, supposing this answer to be involved, we say, what will all of these lbs. cost on the right, if 1000 lbs. opposite, cost 14 mills? Here, the answer is mills, because the last on the right is mills. It would be in cents, if the price of the 1000 lbs. were in cents. Hence, one figure is cut off for mills, and the answer is \$3, 92 cents, and no mills.

What must be paid for the transportation of 18700 lbs. bacon, 60 miles, at 15 mills per 1000 lbs.?

Here, we may place the price in mills, and the answer will be mills; or, \$16,83, no mills. The same result might be obtained by calling the 15 mills  $1\frac{1}{2}$  cents, and placing it down as  $\frac{3}{2}$ , thus,

$$\begin{array}{r|l} & 60 \\ 1000 & 18700 \\ & 15 \\ \hline & \$16,83,0 \end{array}$$

Here, the answer is in cents, and we cut off accordingly. It is optional with the operator whether he place the price in mills or cents: but if in the former, he must cut off, in all cases, one figure at the right, for mills.

$$\begin{array}{r|l} & 60-3 \\ 1000 & 18700 \\ & 23 \\ \hline & \$16,83 \end{array}$$

What are the tolls on 800 bbls. flour, 80 miles, at  $1\frac{1}{2}$  cents per barrel?

$$\begin{array}{r|l} & 80-4 \\ & 800 \\ & 23 \\ \hline & \$960,00 \end{array}$$

What tolls on 20000 lbs. salt, 63 miles, at 1 cent per mile?

$$\begin{array}{r|l} & 63 \\ 1000 & 20,000 \\ & 1 \\ \hline & \$12,60 \end{array}$$

What tolls on 192000 shingles, at 5 mills per 1000, for 20 miles? The answer is 19 dollars and 20 cents.

$$\begin{array}{r|l} & 20 \\ 1000 & 192000 \\ & 21 \\ \hline & \$19,20 \end{array}$$

What tolls on 1200 cubic feet undressed stone, 40 miles, at 4 mills per perch, of 25 cubic feet.

$$\begin{array}{r|l} & 40 \\ 25 & 1200-4 \\ & 4 \\ \hline & \$7,68,0 \end{array}$$

$$\begin{array}{r|l}
 1000 & 15-9 \\
 3000 & \\
 \hline
 23 & \\
 \hline
 & \$8,10
 \end{array}$$

What are the tolls on 3000 cubic feet of lumber, at  $1\frac{1}{2}$  cents per 100 feet, for 18 miles?

What are the tolls on a pile of wood, 200 feet long, 40 feet wide, and 8 feet high, for 60 miles, at  $1\frac{1}{2}$  cents per 100 feet?

$$\begin{array}{r|l}
 1000 & 60 \\
 2000 & \\
 \hline
 40 & \\
 \hline
 8 & \\
 \hline
 23 & \\
 \hline
 & 576,00
 \end{array}$$

Here, it is unnecessary to multiply together the 3 dimensions of the pile of wood: while the placing of the several numbers on the right, will indicate the same. We might call these three dimensions 64000 feet of wood, and use, on the right, only 60, 64000, and  $\frac{3}{2}$ , which would produce the same result.

It frequently becomes necessary to reckon tolls, when it is not convenient to go through with tedious multiplications and divisions: hence, the necessity of combining several operations, as above, in one statement. From the foregoing we conclude, that,

*To ascertain tolls on merchandise, Place the distance in miles, and quantity of freight in lbs., feet, &c., with the price, on the right; and the standard measure, of the specified article of freight, on the left: that is, if tolls are charged per 1000 lbs., place 1000 on the left: if on 100 feet, place 100 on the left, &c.*

## COMPOUND PROPORTION.

We have seen that a simple proportion is formed by the combination of two equal ratios; hence, a Compound Proportion is formed of two or more of these simple proportions, instead of a compound and simple ratio, as is said by some authors. In some cases, as in the following example, the compound proportion is formed of the two combined ratios of the causes, and the one ratio between the effects; but when there are more terms in the effect than one, it is found necessary to ascertain a number of ratios, as well in the effects, as in the causes; so that in such case, the compound proportion would be composed of two compound ratios, instead of one compound and one simple.

It may be shown, as in the question following, that by two or more statements in simple proportion, we may easily ascertain the result of a compound question.

If 4 men in 8 days mow 12 acres of grass, how many acres will 8 men mow in 16 days? The first statement is, as 4 men to 8 men, so are 12 acres to 24 acres: the second, as 8 days to 16 days, so 24 acres to 48 acres. Thus, how many acres will 8 men mow, if 4 men mow 12 acres?

The answer is 24 acres.

$$\begin{array}{r} 4 \text{ } \text{---} 2 \\ 12 \\ \hline 24 \end{array}$$

Again: How many acres will 16 days work give, if 8 days work give 24 acres?

The answer, 48 acres, is thus obtained by two separate and easy statements. Here it is seen, that in each of the two ratios, we have made a proportion; which proportion has in each case shown an increase in the number of

$$\begin{array}{r} 8 \text{ } \text{---} 2 \\ 24 \\ \hline 48 \end{array}$$

acres, until this number has become as great as if originally multiplied by the two ratios combined, which make 4; thus,  $4 : 8$  gives 2, and  $8 : 16$  gives 2, and twice 2 are 4, the combined ratios. Now the 12 acres multiplied by this ratio 4, becomes 48 acres, as before.

In compound proportion there are always five terms given to find the sixth, seven to find an eighth, nine to find a tenth, eleven to find a twelfth, thirteen to find a fourteenth, fifteen to find a sixteenth, etc., etc.

In a compound, as in a simple proportion, the two mean terms are always equal to the two extremes. Because there may be ten, twelve, or more terms in a compound proportion question, is no reason that the theory and the governing principles of the question are any the less the doctrine of means and extremes, than if there are four terms only. All of these several terms must be so classified that they can be united in four distinct bodies; which four bodies become means and extremes of proportion. The great difficulty with most authors on arithmetic has been to systematize this classification, so as to present it as a general truth. This classification depends on the relations of *cause* and *effect*; and although many years since a European writer, Dr. Lardner, discovered that these causes and effects were the great *foci* in the statement of such questions, yet no system of bringing them together, or into their appropriate connection and place, was discovered, by which their use could be availed. It has been so in all sciences. Many brilliant practical and useful axioms have been discovered, that have lain dormant in neglect, merely because the *modus operandi* of their application did not accompany the discovery, so as to give it efficiency.

We shall, therefore, consider,

FIRST—*The principles involved in Cause and Effect:*

SECOND—*The application of these principles; and,*

THIRD—*The form of statement according to the necessities of their relation, so as properly to avail all the benefits of cancelation in their reduction.*

Causes are anything that involve *action*, or imply *capacity*; and which in their action, or the contents of their capacity, produce effects. *Action* always commences in a life-giving principle; pursues some regular medium; and invariably shows some *effect* when it ends. *Capacity*, or *Geometrical extent*, instead of producing any effect, merely exercises an influence over effects. It produces nothing, because it has no action; this action being always essential to the formation of an object. *Geometrical extent* pertains to containing, circumscribing, or consuming effects; and, as such, is not an *active*, but a *passive* cause.

The principles in Cause and Effect pertain strictly to matter; and, as such, admit of no subdivision. They form simply the two categories of the *producing* and the *produced*: the *agent* and the *object*.

The medium through which causes operate to produce effects cannot be real or material; has no parts, and is only imaginary. It is like a ray of light shooting through the aperture of a window of a darkened room, and leaving a brilliant spot upon the wall. The bright spot is the *effect*; the sun, the *cause*; the space between them being only a straight line, conceivable and imaginary, though without parts. A house that has been built by a man is the effect, while the man is the cause. No relation exists between these but the *builder* and the *built*. It may be urged that there are instrumentalities necessary to enable the cause or agent to do this work or produce this effect. A proper analysis will show, however, that these instrumentalities should be merged into their prime causes, and become part of them. It would be said, that in

building the house, *tools* were the instruments; but a little reflection will suggest that these are used only to give the *hands* more efficiency, and facilitate the execution of the work. So the truth is resolved back, that the man is the plastic cause that conforms and molds to his own taste; and the house, the object made.

Ancient philosophers divided and subdivided these agencies and effects, in actual and material operations, into ten or twelve different categories; but these are both useless and unreasonable, and tend greatly to confusion; while the practical fact recurs, that all objects in nature are directly, either causes or effects.

Causes implying *action* as well as *capacity*, and causes definitively speaking, being active and endowed with life, we conclude, that

FIRST—*All animate things are causes.* Hence we say, *active causes are men and animals.* When we say men, we include the whole human race—*men, women, and children.* These constitute a higher order of causes, being endowed with reason; and may, therefore, be called *intelligent causes.* When we say animals, we refer to every *living, moving, creeping, flying thing* that God has made. Such may be classed among *active irrational causes.*

Capacity being considered cause, we conclude that

SECOND—*All time is cause;* that is, every conceivable subdivision of time, as centuries, generations, years, months, days, hours, minutes, seconds. Time within itself has none of those active powers that can entitle it to the name of an *efficient cause*; but it seems to be co-efficient, from the fact that it gives extent or capacity in which *active and operative causes* may produce effects. This capacity, or sphere of action, it will be shown, is as essential to the existence of the effect as the cause itself; though in a secondary degree in point of production.

Geometrical extent is not unfrequently the bounds determining the extent of matter consumed or used; and hence the occupation of the vacuum or limit, depends on its prescribed limits. Hence we conclude, that,

THIRD—*Geometrical extent is sometimes a cause, and sometimes an effect: a cause when no more ostensible or efficient cause is found in the question; and an effect when the dimension of something produced by an efficient cause.*

For instance: it is a *cause* when, as in making cloth, etc., a certain number of yards in length and quarters in width consume a given quantity of wool: it is an *effect* only when the dimension of something produced; as the length, width, and height, of a wall, etc., built by a given number of men, etc. In the latter case, it is an effect, because the work is effected by active causes, such as men. As the dimensions of a wall, this extent *might* be a cause; as in determining the quantity of stone consumed in its construction.

Anything representing active endeavor may be called a cause: hence we conclude, that,

FOURTH—*Capital is a cause, when it produces interest*; because it has delegated to it all the productive and cumulative capacities of active individual effort. Capital may be said to be a cause only when it produces interest. There are, however, instances in which a sum of money becomes a cause; but only on similar grounds to that of gaining interest, as the representative of active powers. It is only in transferring action to the object that the object becomes a cause. When this is done by conventional usage, as in the case of interest, where the laws give this quality to capital, it may be called a *relative cause*.

It is a self-evident truth in common sense, that a certain number of separate causes must produce the

same number of separate effects: or that the extent of the effect is greater or less, as the cause is greater or less; and that the quality of the effect depends on the nature of the cause; or as the change is, in the cause, so must it be in the effect.

That causes always exist before effects are produced; and that no effect can be produced without a commensurate cause; and that causes, *as* causes, always produce effects, are alike self-evident and reasonable.

The cause occupying one position and the effect another, thus,

CAUSE—————EFFECT,

and the space between them being an imaginary line, no one can doubt that they are as directly opposite in their *nature* and *relations* as east and west; right and left. Nor will it be disputed, that if from the cause we pursue a given direction to find the effect, we must, in returning from the effect to the cause, trace back the line by which we first sought the effect.

We know that the united energies of any number of causes, cannot in a natural operation produce more than one effect at a time: otherwise it would be necessary for them to operate in two directions at the same time, which would be impossible; for as unity of cause produces *one* thing, so, whatever produces *more* than any one thing at the same time, must be more than one train of causes. But, as no object can be encompassed or circumscribed by the *direct* operation of any one other object, influence, cause, or thing, so no one cause, or agent in the sum of cause, can, unaided by another agent, produce an effect.

Although *men* may be considered one of the most active species of causes constituting the sum of causation, yet, the 8 men, above would never, of themselves, mow 12 acres of grass; for, however active they may be

as one element of causation, yet their powers must have *some sphere of action*; and this sphere is *time*. So that when we combine the energies of the men with the capacity of the time, both together constituting a *community of endeavor*, are enabled to encompass or effect the desired object. *Any one thing by itself as a unit, is inoperative; but when multiplied into something else, cumulates or expands in the ratio of squares.* It is this expansion of the combined energies of causes that enables them to encompass effects; and it is the necessity of this, which prevents any one bare agent in a cause producing any effect by itself.

It may be said that certain chemicals produce powerful effects by their lone agency: this, however, cannot prove the position, that any one essential element of cause can produce an effect; for whenever any chemical effect is produced, it is the consequence of the *combination of different essential elements, within the chemicals, or in the air, which is the medium of operation.* Hence, different effects are produced by the combination of different elements; the combined effort being the parent of the inception.

From the foregoing we may safely conclude, that  
*Causes are*

MEN,  
ANIMALS,  
TIME,  
CAPITAL, OR  
MEDIUM:

*Or, whatever produces an effect.*

Every problem in Compound Proportion, and in Simple Inverse Proportion, is composed of its terms of supposition, and its terms of demand; and every supposition and every demand has in each, *causes and effects.*

These causes and effects we will endeavor to classify, so as to form a rational and philosophical state-

ment, which will, at the same time that it is clear, be unencumbered with those varied immanageable dependencies of terms, common to the old form of statement; while it will be general in its application, and susceptible of all the abbreviation practicable in cancelation. Some of the old works have indeed canceled some little in this department of arithmetic; but not to any very useful extent, by reason of having no system of arranging all of the terms so as to assume the form of *dividend* and *divisor*. This is the great desideratum, so far as canceling is concerned, in all modern works that pretend to use it, in any other, as well as in this department of numbers.

The student has to analyze and state a question thoroughly, before he can determine what terms are divisors and what dividends. He can have, therefore, no regular or unique system of statement; and is at all times necessitated to hunt up *dividends* and *divisors* merely *as* such; whereas, by this method, he has the well-defined landmarks of *cause* and *effect*, and states according to the *two* great and infallible directions that nature has given him, in all created things.

In the following problem we will assign to *causes* and *effects* their appropriate places.

*If 4 men in 8 days mow 12 acres of grass, how many acres will 8 men mow in 16 days?*

We write the supposition first, and then the demand, and draw a line under each; thus,

men. days. acres.					men. days. acres.			
4	..	8	..	12	—	8	..	16 .. 0
<hr/>						<hr/>		

A short line is likewise drawn under the causes both in the supposition and demand. Here 4 men and 8 days are the causes in the supposition, and 12 acres the effect: in the demand, 8 men and 16 days are the



tors of each cause on the line, we may place their product; thus,

32|12 128|0

Here it is perceived, that we have three terms of a proportion given to find the fourth; that is, *the two means and one extreme, to find the other extreme*. Now, we know that if these two means be multiplied together and divided by the given extreme, the other extreme will be found; which, too, will be the required answer. The question, as now stated, appears to be, *as the cause, 32, is to the cause 128, so is the effect, 12, to the required effect, 48*; or, *as the first cause is to the first effect, so is the second cause to the second or required effect*.\*

This proportion may be transposed as other proportions, into eight different forms or readings; the causes and effects being the terms. When either the *means* or *extremes* are multiplied together, they are the product of one cause and one effect; and this product is divided by the given cause, to find the required effect; or by the given effect, to find the required cause.

But suppose this question be changed and proven, by saying: If 4 men in 8 days mow 12 acres of grass, in how many days will 8 men mow 48 acres? Here, 4 men and 8 days are causes in the supposition, and 12 acres the effect: 8 men and blank (0) days the causes in the demand, and 48 acres the effect; thus,

4|12 8|48  
8| 0|

We perceive that it takes both men and days, two causes, in the supposition, to produce the effect, 12 acres; and infer that two similar causes should exist in the demand, to produce the one effect, 48 acres. In the de-

\* We do not allude here to the specific ratio between the causes and their effects; we only refer to their categorical and regular bearings. Strictly speaking, there is no ratio between cause and effect, except that accidental ratio which depends upon the nature of things, in their unclassified and irregular state. We desire to discard this forced, and irregular, and circumstantial connection or ratio, and place proportion on its true basis,—*ratio* between things that are alike.

mand only one of these causes is found; hence, the cause is deficient; that is, one of the terms of the means is deficient; and a member that is necessary to constitute the causation, producing the effect, 48. Hence, while all of the extremes are multiplied together for a dividend, the product should be divided by all of the remaining means to get the required mean. We know that 16 days is the answer required. Now, the product of the extremes is  $4 \times 8 \times 48 = 1536$ , which, divided by the product of the means  $12 \times 8 = 96$ , gives 16, the number of days required. Thus, we might drop any one of the terms, of either the means or extremes, and ascertain it, by dividing the product of the other two, by the remaining terms.

The deficient term, or that in which the answer is required, is always either a cause or an effect. This deficiency must always be indicated by a *cipher*. It never falls on either side of the left, or line of supposition; because the supposition must in all cases be perfect, and have, consequently, no deficiency; but it is always found on either the right or left side of the right line, as the term wanting may be an effect or a cause; for this demand is always deficient; and it is this deficiency which *makes* it a demand, by asking what will supply it; while the operation is performed for no other purpose than to supply it.

To be certain that the blank, or cipher indicating the deficiency, is located correctly, we must consider whether the answer desired is a cause or an effect. If a cause, the blank is placed under the head of cause, on the *left* side of the line; if an effect, it must be placed under the head of effect, on the *right* side of the right line. Or we may count the number of causes on each line, and if they be equal, the deficiency is an effect. Again: we may count the number of effects on each line; if they are not equal, the deficiency is found; if they are equal, the deficiency must be among the causes.

After placing all of the terms in the question on the two lines, according to the directions given above, notice whether the blank falls among the mean terms; that is, between the two lines; or whether among the extremes, or terms outside the two lines: if it fall among the inner terms, all of the inner terms must be placed on the left side of the vertical line, or line of ratio, on which the question is to be wrought: if among the outer terms, all of the *outer* terms must be placed on the left of this line. This is based on the principle of dividing by the mean or extreme, as the one or the other may be deficient, to ascertain the deficient term. Being thus stated, the question may be canceled as in other cases. Let us now recur to the first question:

If 4 men in 8 days mow 12 acres, how many acres will 8 men mow in 16 days? and state it thus,

$$\begin{array}{r|l} 4 & 12 \quad 8 \quad 0 \\ 8 & \quad \quad 16 \end{array}$$

Here we inclose the mean terms entirely, which are 12, 8, 16, that they may be clearly distinguished from the extreme, which are 4 and 8. The blank falling on the outside, the outer terms become the divisors, and the inner terms the dividend, and are placed on the right; thus,

$$\begin{array}{r|l} 4 & 12-3 \\ 8 & 8 \\ \hline & 16 \\ \hline & 48 \text{ acres.} \end{array}$$

Eight equals 8; 4 into 12, 3 times, and  $3 \times 16 = 48$  acres, the answer.

We now resume the second question, which will prove this correct.

If 4 men in 8 days mow 12 acres of grass, in how many days will 8 men mow 48 acres?

Four and 8 are the causes, in the supposition, and 12 the effect; 8 and blank the causes, in the demand, and 48 acres the effect. After stating the question, thus,

4|12 8|48 it is seen that the blank is among the means; hence, the means, 12 and 8, become the divisors, and are accordingly placed on the left; while 4, 8, and 48, the extremes, are placed on the right of the line, where the dividend is always placed; thus,

$$\begin{array}{r|l} \cancel{8} 4 & \\ \cancel{12} \cancel{8} & \\ \hline 4 \cancel{8} - 4 & \text{Days being the deficient term, the answer is 16 days.} \\ \hline 16 \text{ da.} & \end{array}$$

Again: If 4 men in 8 days mow 12 acres, how many men will be required to mow 48 acres in 16 days?

$$\begin{array}{r|l} 4|12 & 0|48 \\ 8| & 16| \end{array} \quad \begin{array}{r|l} \cancel{12} 4 - 2 & \\ \cancel{8} - \cancel{16} 8 & \\ \hline 4 \cancel{8} - 4 & \\ \hline 8 \text{ men.} & \end{array}$$

Here, again, the mean terms become the divisors. Hence, 8 men, the answer.

Again: changing the supposition and demand, If 8 men in 16 days mow 48 acres, how many acres will 4 men mow in 8 days?

$$\begin{array}{r|l} 8|48 & 4|0 \\ 16| & 8| \end{array} \quad \begin{array}{r|l} \cancel{8} 4 \cancel{8} - 12 & \\ \cancel{4} - \cancel{16} 4 & \\ \hline 8 & \\ \hline 12 \text{ acres.} & \end{array}$$

In this instance, one of the terms of supposition is dropped, which makes it deficient; it becomes, consequently, the demand; while all of the terms of the former demand are supplied; making the demand, in its turn, the supposition. Hence, the reason for placing it on the left line. The answer is 12 acres.

Again: If 8 men, in 16 days, mow 48 acres of grass, in how many days will 4 men mow 12 acres? thus,

$$\begin{array}{r|l} 8 & 48 \\ \hline 16 & \end{array}$$

$$\begin{array}{r|l} 4 & 12 \\ \hline 0 & \end{array}$$

$$\begin{array}{r|l} 4 & 8 \\ \hline 4 & 16 \\ \hline 4 & 12 \\ \hline 8 & \text{da.} \end{array}$$

The first question has now been proven in all of its terms; showing conclusively the similarity between the two causes, and the two effects, as the four terms of a proportion.

Having presented the foregoing *rationale* of Compound Proportion, we will now state and solve a number of problems involving all of the varieties common to this department; that in them the reader may have a sure guide to the solution of all similar questions, occurring either in theory or practice.

If 4 men, in 8 days of 10 hours long, mow 40 acres of grass, in how many days of 12 hours long will 15 men mow 60 acres?

$$\begin{array}{r|l} 4 & 40 \\ \hline 8 & \\ \hline 10 & \end{array}$$

$$\begin{array}{r|l} 15 & 60 \\ \hline 0 & \\ \hline 12 & \end{array}$$

$$\begin{array}{r|l} 12 & 4 \\ \hline 3 & 16 \\ \hline 40 & 160 \\ \hline 60 & 120 \\ \hline 2\frac{2}{3} & \text{da.} \end{array}$$

Here, the different lengths of the days are expressed by the hours, as causes, in each case. Hours have, therefore, assigned to them the place of cause, as all other causes of time; and cooperate with the men and days to produce the effect, by extending the time or sphere of execution, and defining its limits. The answer is  $2\frac{2}{3}$  days.

If 4 men in 20 days of 8 hours long, build a wall 400 feet long, 32 ft. wide and 5 ft. high, in how many days, 15 hours long, will 6 men build another wall, 300 ft. long, 80 feet wide, and 30 feet high?

While 4, 2, and 80 are the causes in the supposition, 400, 32, and, 5 are the dimensions of the effect, which, properly considered, is a unit. In both cases, supposition and demand, these dimensions are made the effect, and we proceed with the work as hereinbe-

fore described. The answer being required in days the blank comes under the head of cause, to show it. Hence the mean terms are the divisors, and the answer is a term of cause, 80 days.

4 400	6 300	5—15 4
20 32	0 80	8 20
8 5	15 30	400 8
		22 200
		8 80
		20
		80

Two ciphers equal two ciphers; 8 times 4 equal 32; 6 times 5 equal 30; 3 into 15, 5, and 5 times 4 equal 20; the answer is 80 days. Now, let us prove this question by using this answer, and dropping some other term; the 80 feet wide, for instance. The question will be as follows:

If 4 men in 20 days of 8 hours long, build a wall, 400 feet long, 32 feet wide, and 5 feet high, how wide will that wall be, the length being 300 feet and the height 30 feet, which 6 men, in 80 days, 16 hours long, will build?

4 400	6 300	300 400
20 32	80 0	20 32
8 5	16 30	4 5
		20 6
		8 80
		16—8
		85½ ft. w.

Here it is seen that the blank comes under the head of effect, one of the dimensions of the wall, which causes us to divide by the extremes.

The following question involving fractions may be as easily disposed of as the others. We will simply place the mixed numbers on the two lines of supposition and demand, without reducing them, until they are brought to the line of ratio; when, after being reduced as in other cases, to improper fractions, the numerators will occupy just such position as if they were whole numbers, with their respective denominators opposite them. Thus, the question will be

reduced to the form of whole numbers, avoiding the numerous fractional difficulties which usually render such solutions too complex to be interesting to the general reader.

If 5 men, in  $7\frac{1}{2}$  days of  $12\frac{1}{2}$  hours long, dig a ditch 400 feet long,  $4\frac{1}{2}$  feet wide, and  $6\frac{1}{4}$  feet deep, how many men will be required in 20 days,  $6\frac{1}{2}$  hours long, to dig another ditch 640 feet long,  $3\frac{1}{2}$  feet wide, and  $3\frac{1}{2}$  feet wide?

$$\begin{array}{r|l} 5 & 500 \\ 7\frac{1}{2} & 4\frac{1}{2} \\ 12\frac{1}{2} & 6\frac{1}{4} \end{array} \quad \begin{array}{r|l} 0 & 640 \\ 20 & 3\frac{1}{2} \\ 6\frac{1}{2} & 3\frac{1}{2} \end{array}$$

We have placed all of the inner terms on the left, and all of the outer terms on the right, without observing any particular order as to which should be placed on the line first.

$$\begin{array}{r} 20 \cancel{640} - 2 \\ 22\frac{1}{2} \\ 500\frac{1}{2} \\ 8 - 22\frac{1}{2} \\ 25\frac{1}{2} \\ 21\frac{1}{2} - 5 \\ 22\frac{1}{2} \\ 115 \\ 51\frac{1}{2} \\ \hline 815 \\ \hline \text{Men. } 17 \end{array}$$

If 6 men in 10 days, 8 hours long, dig a canal 200 yards long, 80 yards wide, and 3 yards deep, in how many days of 12 hours long, will 5 men dig another canal 300 yards long, 40 yards wide, and 16 yards deep, supposing the difficulty or density of the soil to be, in the latter case to the former, as 5 to 2?

It is palpable, that 2 degrees density of soil must be placed, among the effects of the supposition, to determine the extent or difficulty of the effect to be produced; while, for the same cause, 5 degrees of density are placed among the effects of the demand. The same results could be obtained, if  $2\frac{1}{2}$  were placed among the effects of the demand, or  $\frac{2}{3}$  among those of the supposition; in this instance, however, the symmetry of the terms would be lost, and the statement is made according to the first suggestion; thus,

6 200	5 300	5 6
10 80	0 40	12 10
8 3	12 16	8
2	5	200 300

We find in Compound Proportion many conditions that can be appended to questions, which ordinarily render them apparently very complex; but these conditions may be disposed in the most summary and natural manner, by assigning to the terms representing them, their proper places as causes and effects.

How long will 8100 lbs. of bread serve a garrison of 100 men, if 900 lbs. are consumed in 50 days by 20 men? *If*, in this instance as in all others, indicates the supposition, which is 20 men, 60 days, and 900 lbs. of bread; while the demand is 100 men, 0 days, and 8100 lbs. We state as in other cases; remembering that whatever the language in which a problem is stated, whether the men eat, or whether the bread is consumed,—whether the demand or supposition be given first, we must transpose the question, until it is brought before the mind in its proper bearings; and then state according to common sense. Some would think, at first sight of this question, that because the pounds of bread served the garrison, these pounds must necessarily be the cause, and the garrison the effect. Such must consider, and ask in what term the *greater action exists*; whether in the bread, in supplying or being eaten, or in the men, by eating and consuming it. The latter implies action; while the former is only subject to action, and as an object in its passive form, becomes an effect,

20 900	100 8100	100 20
50	0	800 50
		8100—9
		90 days.

If 60 horses in 20 days consume 15 bushels of oats, how many horses will 600 bushels feed 120 days?

Horses become the deficient term; hence we divide by the inner terms, and find that 400 horses will consume the 600 bushels in 120 days.

$$\begin{array}{r|l}
 60 \overline{) 15} & 0 \overline{) 600} \\
 20 \overline{) 120} & \\
 \hline
 \end{array}
 \quad
 \begin{array}{r|l}
 120 \overline{) 60} & \\
 15 \overline{) 20} & \\
 \hline
 400 &
 \end{array}$$

This question, as well as the foregoing, and all that will follow, may be proven in as many different ways as there are different terms given in the supposition and demand.

If 400 mosquitoes, in 30 nights, 15 hours long, raise on an animal 60,000 lumps,  $\frac{1}{4}$  of an inch in diameter and  $\frac{1}{8}$  inch high, in how many nights, of 10 hours in length, will 800 mosquitoes be required to produce 50,000 lumps,  $\frac{1}{8}$  of an inch in diameter, and  $\frac{1}{16}$  of an inch in height?

Here, the number of lumps, their diameter, and height are the effects, in each case; while the mosquitoes, nights, and hours are the causes. We state accordingly.

$$\begin{array}{r|l}
 400 \overline{) 60,000} & 0 \overline{) 50,000} \\
 30 \overline{) \frac{1}{4}} & 800 \overline{) \frac{1}{8}} \\
 15 \overline{) \frac{1}{8}} & 10 \overline{) \frac{1}{16}} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r|l}
 800 \overline{) 400} & \\
 10 \overline{) 30} & \\
 60,000 \overline{) 15} & \\
 14 & \\
 18 & \\
 50,000 & \\
 169 & \\
 201 & \\
 \hline
 16\frac{1}{2} \text{ ni.} &
 \end{array}$$

In this question numerous other conditions might be added: it might be said that the warmth of the night, in the one case, was to that of the other as a given ratio: also, it might be said, that a certain ratio of difference existed between the two subjects operated on, as to adhesiveness of material, etc. All of these conditions would be legitimate in such questions, at least in theory.

We now come to the consideration of questions in which the causes that produce effects, instead of being

active, are merely passive; with an efficiency delegated, rather by the attendants, and controlling circumstances and customs of the day, than any inherent or vital principle of action.

Capital and time are causes in the production of interest, not because they have any active powers that would enable them to encompass or accomplish this object, but because *public common consent grants that capital may, by representing individual effort, be supposed to do this*; and this supposition or permission of the thing, is equivalent to the act or fact, "other things being equal," so far as matters of common consent are concerned.

If I lend my friend, in his need of money, \$200, for 20 days, and he agrees to render me a similar accommodation when necessary; and when my occasion requires it, he has only \$150, how long must I use this sum to remunerate me for the loan of the \$200?

The question appears to be this: If \$200 in 20 days, render 1 accommodation, in how many days will \$150 render 1 accommodation? The 200 and 20 are the causes in the first case, and 1 accommodation the effect; while, in the other, 150 and 0 days are the causes, and, as before, 1 accommodation the effect. State as follows:

200 1	150 1	150 200
20	0	120
		1
		<hr/>
		26½ days.

Hence the answer,

The pupil will notice, that in the foregoing proposition the effects are equal; that is, the effect in the supposition and demand is the same, or *common*. We shall have occasion to use this fact in our remarks on Inverse Proportion; for it is easily seen, that if the one effect equals the other, they can both, *as terms*, be dispensed with: hence, in the statement of the ques-

tion without them, we would find only three terms given for ascertaining the fourth; and this would make it a simple proportion.

Let a few other questions be now presented, in which capital is cause, and which will teach the pupil the very important method of finding *principal, time, and rate* in interest. We will find, first, the interest on \$200 for 3 months, at \$6 gain on the \$100, or 6 per cent.; and here, as in other cases, we must have a supposition, which, if not given in the question, must be taken from legally-established custom. We say, therefore,

If \$100 dollars, in 12 months, gain \$6 interest, what interest will \$200 gain in 3 months?

The causes in the supposition are \$100 and 12 months, while \$6 interest is the effect; and in the demand \$200 and 3 months, while the effect is wanting. State as follows:

$$\begin{array}{r|l} 100 \overline{) 6} & 200 \overline{) 0} \end{array} \quad \begin{array}{r|l} \cancel{100} \cancel{200} & \cancel{12} \cancel{3} \\ \cancel{12} \cancel{3} & 6 \\ \hline & \$3 \text{ int.} \end{array}$$

We find the sixth term \$3, which is the effect; showing that, if \$100 in 1 year gain \$6 interest, twice this sum, in  $\frac{1}{4}$  the time, will gain  $\frac{1}{4}$  of 6, or \$3 interest.

#### TO FIND THE PRINCIPAL IN INTEREST.

Having ascertained the interest, let us now find the capital that in 3 months would produce this interest. To do this we make the following statement:

If \$100, in 12 months, gain \$6 interest, what sum of capital will gain \$3 interest in 3 months? thus,

$$\begin{array}{r|l} 100 \overline{) 6} & 0 \overline{) 3} \\ 12 \overline{) 3} & 3 \end{array} \quad \begin{array}{r|l} \cancel{12} \cancel{3} & 2 \\ \cancel{3} \cancel{100} & 100 \\ \hline & \$200 \end{array}$$

Our answer is \$200 capital, which is correct; because at the given \$200 prin.

rate per cent. it has just gained, in the time specified, the \$3 interest. Here, it has become necessary to assume the standards of per centum and per annum in interest, for supposition.

TO FIND THE TIME IN INTEREST.

Changing the statement, it may be well to find the time in which \$200, at 6 per cent., will gain \$3 interest: and when we say *at 6 per cent.*, we assume that \$100, in 12 months, 360 days, or 1 year, gains this \$6. Thus, the statement:

If \$100, in 12 months, gain \$6 interest, in what time will \$200 gain \$3 interest? thus,

100 6	0 3	200 100
12	200	6 12
		3
		3 mos.

The answer is 3 months time.

This answer may again be changed so as

TO FIND THE RATE OF INTEREST.

It is only necessary now, to complete this series of proofs and proportions, to ascertain the rate at which money is lent, when a certain sum has gained another specific sum of interest. We commenced in the question, by saying, that the rate per cent. was 6; and have used this rate in the statement of the supposition with which it is connected. It is now desired to find this 6 per cent., per annum. When we say 6 *per cent.*, *per annum*, we mean 6 on the *one hundred*, for or by the *one year*, or 12 months. It cannot be denied, that this \$6 was gained by the \$100 in one year, or 12 months. The former supposition, "If 100 in 12 gain 6," now becomes the demand; one of

its terms, the 6, being deficient. The former demand, therefore, now becomes the supposition, and is as follows:

If \$200, in 3 months, gain \$3 interest, what interest will \$100, or per centum, gain in 12 months, or per annum?

The answer will certainly be the sum that the \$100 in 1 year would gain; that is, \$6, which proves it 6 per cent. per annum.

200 3	100 0	200 100
8	12	3 12-2
		3
		6 per ct.

We may in a similar manner find the rate at which interest has been gained on a given principal, and for any given time, by saying, if such principal, in such time, gain so much interest, what interest will \$100 principal gain in 1 year, 12 months, or 360 or 365 days, as the case may be; for, if the time in the supposition be in years, 1 year must be used in the demand: if in months, 12 months in the demand; and if in days, 360 days. For example:

If \$60, in 120 days, gain \$1½ interest, what is the rate per cent. per annum; or, what will \$100 gain in 360 days? Thus,

60 1½	100 0	60 100
120	360	120 360-3-2
		\$ 6
		6 per cent.

In the question preceding this, the reader will perceive, that although this is a *compound* statement, yet the supposition and demand are located, as in simple proportion, with the term of answer, \$3 interest, last on the right.

This is not an inverse statement, because the answer is required in interest, which is an effect.

Again: Suppose I have a debt of \$40 to pay in 80

days, and have no means of getting the money except by lending to some individual a sum of my capital, large enough to gain this sum of debt, in the given time, at 6 per cent.; how much capital shall I thus put on interest, for the 80 days, to gain the \$40? The question would be stated, thus,

If \$100, in 360 days, gain \$6 interest, what principal will gain \$40 in 80 days?

$$\begin{array}{r|l} 100 \overline{) 6} & 0 \overline{) 40} \\ 360 & 80 \end{array} \quad \begin{array}{r} \$0 \overline{) \$60-6} \\ \$100 \\ \underline{40-5} \end{array}$$

It is thus seen, that the sum to be negotiated is \$3000, which may be easily proven, by asking the interest, at 6 per cent., on this sum, for 80 days, which would be found \$40.

We next proceed to the consideration of *causes of capacity*, properly so called. It is known that the dimensions 8 feet long, 4 feet wide, and 4 feet high make 1 cord. Now,

If 4 feet high, 4 feet wide, and 8 feet long make 1 cord of wood, how many cords will a pile 400 feet long, 60 feet wide, and 16 feet high make?

In each case the *dimensions* are the causes, and the number of cords which they make, the effect. We state accordingly,

$$\begin{array}{r|l} 4 \overline{) 1} & 400 \overline{) 0} \\ 4 & 60 \\ 8 & 16 \end{array} \quad \begin{array}{r} 4 \overline{) 400-5} \\ 400 \\ \underline{60} \\ 60 \\ \underline{16} \end{array}$$

The answer is 3000 cords of wood. 3000 cor.

Now, suppose we wish a pile to be made large enough to make a certain number of cords, where two of its dimensions are given; for example,

If 4 feet wide, 4 feet high, and 8 feet long make 1 cord, how high must a pile be, whose width is 8 feet and length 40 feet, to make 10 cords? thus,

4 1	0 10	8 10
4	8	40 4
8	40	4

The pile must be 4 feet high; which may be easily proven by suspending some other dimension, or finding the number of cords that these complete dimensions will make.

If 10 yards of cloth,  $\frac{3}{4}$  of a yard wide, consume 40 lbs. of wool in the facture, how many pounds will it take to make 20 yards,  $\frac{1}{4}$  of a yard wide?

We have shown that no one cause, by itself, can produce an effect, until united with some other cause; we have likewise shown, that no cause, or combination of causes, can produce more than one effect. Now, allowing the existence of these truths, it would be ridiculous to say that the pounds of wool, in the question above, was the cause of producing so much cloth, so, and so wide. But, to say that the length and width in yards and quarters, disposed of, or consumed a given number of pounds, would be both reasonable and natural. The want of a little discrimination and reflection here, will involve the student in innumerable absurdities; while, on the other hand, a proper and intelligent attention to the relations and operations of causes and effects, cannot fail conducting him to proper and palpable results.

10 40	20 0	10 40
$\frac{3}{4}$	$\frac{1}{4}$	3 4

Our answer,  $93\frac{1}{2}$  lbs., is the effect of the capacity of the 20 yards long and  $\frac{1}{4}$  wide. These two, as causes of capacity, consume or appropriate this number of pounds of wool, as the effect. We are apt to fall into the same unheeded absurdity in questions like the following:

20
8 7
93 $\frac{1}{2}$ lbs.

# COMPOUND PROPORTION : CAUSES OF CAPACITY. 147

If the transportation of 20 tons of freight, 30 miles, cost \$60, how many tons can be carried 800 miles for \$250?

Here the tons and distance are the causes, and the price the effect: thus,

20 60	0 250	800 20
80	800	60 20
		250
		3 1/2 tons.

The answer is 3 1/2 tons.

The same result may be obtained, though improperly, by saying that the dollars are the causes, and the weight and distance the effect. If all the relations of the more palpable operations of cause and effect have but one tendency, and which reversed, disorders the whole, certainly, because in other cases these laws and relations are not so easily perceived, is no reason that they are not founded upon, or governed by, the same fixed and eternal principles. Nor can these principles be neglected or perverted, without consequent disorder, at some time, and in some place.

If \$40 pay for hauling 16 cwt. of hemp 28 miles, how many miles can 56 cwt. be hauled for \$200?

16 40	0 200	56 200
28	56	40 20
		200
		40 miles.

It is an easy matter, in any question, to determine what terms have such a connection as to require that their energies be combined; while it is not less easily determined which term appears the great focus of all these operations. The former are causes, while the latter is an effect.

If a cistern 8 ft. deep, 20 ft. long, and 3 feet wide, hold 340 barrels, how many barrels will be contained in one 28 ft. deep, 60 ft. long, and 15 ft. wide?

The barrels are, in each case, the effects.

8 340	28 0	2-8 340-17
20	60	<del>20</del> 28-7
3	15	<del>2</del> 60
		15

It may be observed here, that in all cases, such as the one above, if inches be annexed to, or connected with, the feet, they must be reduced to the fraction of a foot. Three feet 4 inches may be reduced by saying, 4 inches are  $\frac{1}{3}$  or  $\frac{1}{3}$  of a foot; this fraction added, makes  $4\frac{1}{3}$  or  $\frac{13}{3}$  feet.

If 2150 inches make 1 bushel, how high must a crib be, to contain 800 bushels, which is 200 inches long and 86 inches wide?

In this question, it is the geometrical extent of all the sides of the bushel measure, which is the cause; the 2150 being the product of the three dimensions. The cube root of this number would be one of the sides; and this, placed down three times, on the left side of the line of supposition, would constitute the causes producing the effect, one bushel. As it is unnecessary to find this side and use it three times, which, at best, could only reproduce the number 2150, we merely write the 2150, which is the product of these sides, and produces precisely the same result. In place of the two remaining factors constituting 2150, we use two inverted commas; thus,

2150 1	200 800	200 800-2
“	86	<del>20</del> 86-5
“	0	2150-5
		100 in. hi.

It is seen, that the crib should be 100 inches high. By this mode of operation, either the contents, or any of the sides of a crib, body, box, etc., may be found.

The question above may be proven, by asking, how many bushels a crib will contain, which is 200 inches long, 86 inches wide, and 100 inches deep. Again:

If 2150 inches make 1 bushel, how long must a box

be, to contain 1 bushel, which is 20 inches wide and 15 inches deep?

2150 1	0 1	20 2150-43
"	20	3-15
"	15	648
		7½ in.

We find that it must be 7½ inches long. One bushel is the common effect in both the supposition and demand. We see by this calculation, that a box 7½ in. deep, 20 in. long, and 15 in. wide, will contain one bushel. This may be proven, by multiplying the three sides to a continued product.

We might, in this instance, give a short, but somewhat mechanical, rule for finding any remaining side of any figure of 6 sides, when the contents were a unit of any given standard, or any number of units of such standard, as 1 bushel, or 80 bushels; 1 gallon, or 20 gallons, etc.

According to the foregoing, we may, in a similar manner, find the dimensions of a box or cistern that will hold a given number of gallons, if the standard of unity be first assumed, and used as a supposition; thus:

If 231 inches make a wine gallon, how long, wide, or deep must a box be, to hold any specific number of gallons, if it be of a given width, or depth, etc.?

Beer gallons may become the standard of measure, by using 282, instead of 231 inches

If 231 inches make 1 wine gallon, how long must a box be, to contain 100 gallons, which is 15 in. wide, and 11 in. deep?

231 1	0 100	2-15 231-21-7
"	15	11 100-2
"	11	140 in. long.

By this, it is seen, that the box must be 140 inches long.

If 2688 inches make 1 dry bushel, how wide must a body be, to hold 100 bushels of coal, which is 120 in. long, and 32 in. deep?

Here, 2688 is the product of the three sides of the dry bushel; and, consequently, must be considered as the unit of cause, that produces or contains the 1 bushel. We state as in the similar case above.

2688	1	0	100	<del>120</del>	100
"		120		32	<del>2688</del> —224
"		32			70 in. long.

The width of the body must be 70 inches.

We know that 1 inch wide, 1 inch thick, and  $3\frac{2}{3}$  inches long, make 1 pound of cast iron. Now, these dimensions are the causes in the supposition, and 1 pound, the effect.

We may take the  $3\frac{2}{3}$ , or, which is equivalent,  $3\frac{1}{2}$  solid inches to represent the dimensions of the quantity, and say,

If  $3\frac{1}{2}$  inches make 1 lb. of iron, how many lbs. will a bar make, that is 24 in. long, 4 in. thick, and 6 in. wide?

$3\frac{1}{2}$	1	24	0	<del>4—66</del>	25
		6			24
		4			6
					<del>4</del>
					150 lb.

This  $3\frac{1}{2}$  can be very easily used without the least difficulty in the fraction, as  $\frac{3}{2}$ , as above.

The question may now be changed; and having the width and thickness of the bar, let us see how long it must be to weigh 150 pounds.

By careful attention to the statement and solution of this question, any question arising as to the length, width, or thickness of bars of iron, window-weights, etc., etc., may be easily solved. It frequently becomes necessary, after the two dimensions and weight are given, to find the other dimension, which is

often very troublesome to practical men; especially if many fractions are found in the work. The question is,

If  $3\frac{1}{2}$  inches make 1 lb., how long must a bar be, that is 6 in. wide, and 4 in. thick, to make 150 lbs?

$$\begin{array}{r|l} 3\frac{1}{2} & 1 \\ \hline & 0|150 \\ & 6 \\ & 4 \end{array} \quad \begin{array}{r|l} & 6|150 \\ & 4 \\ \hline & 24 \end{array}$$

From this work it appears, that the bar must be 24 in. long, which is true; because this length was used to find the weight.

If  $3\frac{1}{2}$  inches make 1 lb., 96 inches will make 25 lbs. The product of 96 is composed of 8 inches long, 4 inches wide, and 3 inches thick; therefore, in stating questions of this nature, it may be said,

If 96 inches make 25 lbs; or, if 8 in. long, 4 wide, and 3 thick make 25 lbs., how long, or thick, or wide must a bar be, with two dimensions given, to make any specific number of pounds?

If 8 in. long, 4 in. wide, and 3 in. thick make 25 lbs., how long must a window-weight be, to make 25 lbs., which is 4 in. wide and  $1\frac{1}{2}$  in. thick?

$$\begin{array}{r|l} 8 & 25 \\ 4 & \\ 3 & \end{array} \quad \begin{array}{r|l} 0 & 25 \\ & 4 \\ & 1\frac{1}{2} \end{array} \quad \begin{array}{r|l} 25 & 25 \\ & 8 \\ & 2 \\ & 4 \\ & 3 \end{array}$$

The bar is 16 in. long. Again: 16 in. long

If 8, 4, and 3 make 25 lbs., how wide must a bar be, to make 25 lbs., which is 12 in. long and  $1\frac{1}{2}$  in. thick?

$$\begin{array}{r|l} 8 & 25 \\ 4 & \\ 3 & \end{array} \quad \begin{array}{r|l} 0 & 25 \\ & 12 \\ & 1\frac{1}{2} \end{array}$$

This mode of stating such questions can be easily used, if any attention be given to the general principles.

$$\begin{array}{r|l}
 12 & 25 \\
 25 & 8 \\
 3 & 2 \\
 4 & \\
 3 & \\
 \hline
 & 5\frac{1}{2} \text{ in. wide.}
 \end{array}$$

All mixed numbers must be reduced to improper fractions, here, as elsewhere.

We will now solve one or two other questions, and close this article by endeavoring to show how such statements may be made on one line, as in Simple Proportion.

If 10 men, in 15 days, 8 hours long, compose a book of 16 sheets, 36 pages on a sheet, 25 lines on a page, and 60 letters in a line, in how many days, 10 hours long, will 40 men compose another book, of 48 sheets, 32 pages on a sheet 50 lines on a page, and 90 letters in a line?

$$\begin{array}{r|l}
 10 & 16 \\
 15 & 36 \\
 8 & 25 \\
 \hline
 & 60
 \end{array}
 \begin{array}{r|l}
 40 & 48 \\
 0 & 32 \\
 10 & 50 \\
 \hline
 & 90
 \end{array}$$

$$\begin{array}{r|l}
 10 & 16 \\
 2-40 & 15-5 \\
 2-16 & 3 \\
 4-36 & 25-24 \\
 25 & 32 \\
 4-60 & 50-2 \\
 \hline
 & 30 \\
 \hline
 & 24 \text{ days.}
 \end{array}$$

The answer is 24 days.

There is a novelty in the statement and solution of this question, arising from the fact that it can be wrought without the use of one figure more than is necessary to state it. Instead of canceling, as above, we may have said, 16 into 32, twice; twice 9 into 36, twice; and twice 4, on the left, equals 8; 10, equals 10; cipher, equals cipher; 25 into 50, twice; 2 into 6, three times; 3 into 15, 5 times; five into 10, twice; and 2 into 48, twenty-four times; the answer, as above.

COMPOUND PROPORTION : SINGLE STATEMENT. 153

One more example is given, to illustrate the method of stating on one line.

If 4 men in 16 days, 12 hours long, compose a book of 14 sheets, 24 pages on a sheet, 44 lines on a page, and 40 letters in a line, in how many days, 8 hours long, will 12 men compose a similar work, of 42 sheets, 16 pages on a sheet, 48 lines on a page, and 55 letters in a line?

4 14	12 42
16 24	0 16
12 44	8 48
40	55

12 4
" 16
8 12
14 42
24 16
44 48
40 55
24 da.

Twelve equals 12; 8 into 16, twice, and twice 4, on the right, make 8, which goes into 40, five times; 5 into 55, eleven; 11 into 44, four times; 4 into 16, four times; 4 into 24, 6 times; 6 into 42, seven times; 7 into 14, twice; and 2 into 48, twenty-four times; the answer.

This question may be stated in the following order:

In how many days, 8 hours long, will 12 men produce a certain effect, if 4 men produce the same in 16 days, 12 hours long?

The answer is desired in *days*; a *cause*. Hence, this term, 16 days, is placed last on the right for the term of answer; while the causes, which coöperate with it to produce the given effect, are placed on the same side of the line, where their energies can be multiplied. Now, these causes being necessarily on the right, the causes in the demand must be placed opposite them, on the left. Hence, in all inverse questions, the demand is placed on the left: thus,

Here, if no effects were to be considered, the answer would certainly be in days: 8 days. Thus, we see, that when the answer desired, is a

Men,	12 4	Men,
Hours,	8 12	Hours,
	" 16	Days,
	8 days.	

cause, the demand, that is, the *causes* of demand, must be placed on the left; and those of the supposition, opposite. Now, if causes and effects are always directly opposite in their nature, the effects in the demand, must be placed on the right; and those of the supposition, opposite; thus,

Sheets,	14	42	Sheets,
Pages,	24	16	Pages,
Lines,	44	48	Lines,
Letters,	40	55	Letters,
<hr/>			
3			

It is seen here, that the effects occupy the reverse side of the line. This is, both because causes and effects are opposite in their nature; and because, when

an answer, as the one above, is desired in an effect, the proportion is direct, requiring that the demand be placed on the right. Thus, 3 is the answer; which, multiplied with the 8, gives 24, as in the former work.

Each term is placed opposite the term of its own kind, to get the ratio. This order of statement is reversed, if the answer is required in an effect. In such case, all of the causes, in the demand, are placed on the right, and those in the supposition, opposite; while all the effects are placed opposite their respective causes.

From the foregoing, we deduce the following

#### SUMMARY OF DIRECTIONS FOR COMPOUND PROPORTION.

*Separate the question into terms of Supposition and Demand: Ascertain which terms, both in the supposition and demand, are Causes, and which are Effects: Draw two vertical lines: Place all of the terms of supposition on the left line; and all of the terms of demand, on the right line: Place causes on the left side of each line, and effects on the right.*

*If the answer be required in a cause, place a cipher on the left side of the right line; if in an effect, place a cipher on the right side of the right line.*

*When the cipher falls between the two lines, make all of the inner terms the divisor, and all of the outer terms the dividend: when it falls outside of the two lines, make all of the outer terms the divisor, and all of the inner terms the dividend.\* The causes are men, animals, time, capital, and medium.*

*To state on one line:*

*When the answer is desired in a cause, place all of the causes in the demand, on the left; and all of the causes in the supposition, on the right; with the respective effects of each, opposite:*

*When the answer is desired in an effect, place all of the causes in the demand, on the right; and all of the causes in the supposition, on the left; with the respective effects of each, opposite.*

### SIMPLE PROPORTION INVERSE.

The consideration of Compound Proportion, leads to that of Simple Inverse Proportion; the great leading principles of both, being cause and effect. We have seen, from the solution of one or two questions in compound proportion, that the effect in a compound question being the same, or a unit, in both supposition and demand, there are but three remaining terms to use. The effect, being unity in each case, we say, that the two terms are *common*; and conclude that the problem, as properly enounced, is composed of causes only. It is evident, that if all of the terms are causes, and that as some one of them must be the denomination of the answer, therefore, this answer must, when obtained, be a cause. And here exists the differ-

\* Recollect that the divisor is always placed on the left side of the line on which the question is wrought, and the dividend, on the right.

ence between direct and inverse proportion. From what has been said of the relations of cause and effect, it is evident, that as causes exist before their effects, so the production of these effects is a *direct* or *first* operation of the cause; hence, it is the province of direct proportion to ascertain effects. If, again, causes and effects possess an opposite nature, it is reasonable to suppose, that when the cause is demanded as an answer, the course pursued to find it, will be a retrograde from the effect, to the cause. Hence, the direct course is broken, and reversed; and the line of operation, which the cause first pursued to find the effect, is retraced from the effect to the cause. This inversion, or turning around, is the basis of the theory of *Inverse Proportion*. The term is derived from the Latin, *inverto*, to turn back.

We may always know that a question is inverse, if there are no given terms of effect, whatsoever, above unity. When the effect is not common, the question is necessarily compound; having more than three terms.

All inverse questions are properly compound proportion, and may be solved most intelligibly and easily, by a statement under this head, according to the great principles just treated. Therefore, when they are reduced to the form of simple proportion, we may reasonably expect an entire change in the statement, from questions ordinarily occurring in this department. The following illustration, according to compound proportion, may be given:

How many men will be required to do a piece of work in 15, which can be done in 24 days, by 5 men?

The enunciation of the question may be changed, thus:

If 5 men in 25 days do one piece of work, how many men will be required to do one other piece in 15 days?

The piece of work is the effect, in each case; a common effect; and is located as follows:

$$\begin{array}{r|l}
 5 \overline{) 1} & 0 \overline{) 1} \\
 24 \overline{) 1} & 15 \overline{) 1}
 \end{array}
 \quad
 \begin{array}{r}
 2 - 15 \overline{) 24} - 8 \\
 \underline{15} \\
 1 \\
 \hline
 8 \text{ men.}
 \end{array}$$

It is seen, that in dividing by the inner terms, the 15 is placed on the left. This term would be the demand in simple proportion; and, it is evident, that it is placed on the left, that, by dividing the product of the other two terms, the fourth term may be found; which, linked with the 15, would complete the proportion. It is not, however, placed on the left *as a factor only*, to find another factor; but that its ratio, with a similar term on the opposite, may be ascertained. The 5 men is the denomination of the answer, and must, consequently, be placed last on the right. Now, this 5 men does not equal in value the term of supposition opposite, the 15 days, which is the sole connection between the two similar terms in direct proportion; but, as a cause, it is combined with another cause, the 24 days, to produce the common effect, one piece of work. That these two causes may occupy a position where their energies may be multiplied together and cumulated, the 24 days is placed on the right of the line, in the place ordinarily assigned to the demand, and just over its coöperative term, 5 men; for causes producing a common effect, cannot be separated. Now, to obtain the ratio between this 24 days in the supposition and the 15 days in the demand, the 15 must be placed opposite. Hence, by the necessity of purely philosophical principles, the demand in inverse proportion is inverted, or placed on the left. This is the only satisfactory reason why the demand changes its place, or is inverted. These laws, necessitating more than one cause to produce an effect, and an actual connection of the various terms employed, so as to elicit their

combined energies, in their creative capacity of some effect, have been so clearly elucidated and demonstrated in the discussion of first principles, as to leave no doubt, on the part of the reader, that both terms in the supposition must be placed on the right; one, as the denomination of the answer, and the other as the term to be compared with the term of similar name, in the demand, on the left. The two ones may be dropped in the statement. Again:

If 15 men, in 8 days, do a piece of work, how many men will do the same in 24 days? thus,

$\begin{array}{r} 8-24 \quad 15-5 \\ \hline 8 \\ \hline 5 \text{ men.} \end{array}$	The demand is placed on the left; the same name opposite it; and the term of answer, last on the right. Let us prove this again.
---	--

If 8 men, in 15 days, do a piece of work, how many days will 5 men be required to do the same?

$\begin{array}{r} 8 \\ 15-8 \\ \hline 24 \text{ days.} \end{array}$	While men are compared in this statement, days become the term of answer.
---	---

It is sometimes difficult to determine what is cause, and what effect; especially, when geometrical extent is a cause. Therefore, for the benefit of those who are not experienced in the designation of causes, we give the following directions for stating such questions correctly; and which, to some minds, is the only means of determining whether a proportion is direct or inverse:

*If the answer desired should be larger than the same term in the question, place on the left the smaller of the two terms to be compared by ratio: if the answer required be smaller, place on the left, for the demand, the larger of the two terms to be compared. This is what is called by the old schoolmen, "more giving less, and less giving more;" which simply*

means, if the one cause or factor in a question be larger, the other must be smaller; and vice versa.

Most authors have written about the necessity of inverting terms in inverse ratio, as they call it (which, in truth never existed), as well as about more giving less, and less giving more: but none of them have ever explained satisfactorily the reason for this inversion, or why "more gives less, and less more." Nor have any of them so reasoned on the assumption, as to give the pupil even their own vague and indefinite ideas; much less satisfying his common sense with a rational and demonstrable theory. No doubt that the best arrangements practicable have been made; for it is wholly impossible to explain the theory on any other principles than those of cause and effect. These are the very embodiments of inverse action; the one going directly on to create; the other returning to the creator or cause, from the created effect. The obedient pupil has too long followed in the wake of rules that linger along in obscurity and darkness, not only in this but in most of the departments of this beautiful science.

How many yards of cloth,  $\frac{4}{5}$  quarters wide, are equal to 40 yards,  $\frac{5}{4}$  quarters wide?

Here,  $\frac{4}{5}$  quarters is the demand,  $\frac{5}{4}$  quarters the same name, and 40 yards the term of answer.

$$\begin{array}{r} 45 \\ 40 \\ \hline 50 \end{array}$$

If it take 20 yards of cloth,  $\frac{3}{4}$  of a yard wide, to make a gown, how many yards,  $\frac{7}{8}$  wide, will it take to line it?

The demand,  $\frac{7}{8}$ , is placed on the left, by its numerator; while the same name,  $\frac{3}{4}$ , is, in the same manner, placed on the right.

$$\begin{array}{r} 7\frac{3}{4}-2 \\ 4\frac{3}{4} \\ 20 \\ \hline 17\frac{1}{4} \end{array}$$

How many yards of silk serge,  $2\frac{1}{4}$  yards wide, will

line a coat, that requires 15 yards of cloth,  $\frac{3}{4}$  of a yard wide?

$$\begin{array}{r|l} \cancel{3}-\cancel{4} & 4 \\ \cancel{4} & 3 \\ \hline & 15-5 \\ & 5 \text{ yds.} \end{array}$$

Fifteen yards being the denomination of the answer, is placed last on the right.

If 5 yards long, and  $2\frac{1}{4}$  yards wide, line a coat, how long must the piece of cloth be, which is  $\frac{3}{4}$  of a yard wide, to make it?

$$\begin{array}{r|l} \cancel{3} & 4 \\ \cancel{4} & 3 \\ \hline & 5 \\ & 15 \text{ yds.} \end{array}$$

The demand is  $\frac{3}{4}$  in this case, while the same name is  $2\frac{1}{4}$ . Again:

If 15 yards long and  $\frac{3}{4}$  wide make a coat, how wide must 5 yards of serge be, to line it?

$$\begin{array}{r|l} \cancel{3} & 15-\cancel{3} \\ \cancel{4} & 3 \\ \hline & 2\frac{1}{4} \end{array}$$

In this case, the two terms to be compared, are in length; while width is the denomination of answer. The serge must be  $2\frac{1}{4}$  yards wide?

If 5 yards long and  $2\frac{1}{4}$  wide, make a coat, how wide must 15 yards in length be, to line it?

$$\begin{array}{r|l} \cancel{3}-\cancel{4} & \cancel{5} \\ & 4 & 3 \\ \hline & 4 & 3 \\ & 4 \end{array}$$

The answer is  $\frac{3}{4}$  of a yard wide.

It is perceived, that these questions are inverse, because all of the terms are causes of capacity.

Suppose a garrison has provisions to last 4 months, at 20 ounces per day; how many ounces should they use, for it to last 6 months?

The two different number of months must here be compared; 6 months being the demand, and 20 ounces the term of answer.

$$\begin{array}{r|l} 8- & 64 \\ \hline & 20 \\ \hline & 13\frac{1}{2} \end{array}$$

If, when wheat is worth 60 cents per bushel, the cent loaf weighs 10 ounces, how much ought it to weigh when wheat is worth \$1.25 per bushel?

The prices are compared, as causes, and the weight is the term of answer.

$$\begin{array}{r|l} 125 & 60 \\ \hline & 10 \\ \hline & 4\frac{1}{2} \end{array}$$

If, when wheat is worth \$1.25 cents, the cent loaf weighs  $4\frac{1}{2}$  ounces, what ought it to weigh when wheat is 60 cents per bushel?

The answer is 10 ounces.

$$\begin{array}{r|l} 60 & 125 \\ \hline & 5\ 24 \\ \hline & 10\ \text{oz.} \end{array}$$

If a bushel of wheat make 40, five cent loaves, how many eight cent loaves will it make?

The ratio is obtained between the 8 and 5 cents, as the two most active causes con-  
ducting to the effect. Thus, *capital is at times a cause.*

$$\begin{array}{r|l} 8 & 5 \\ \hline & 40-5 \\ \hline & 25 \end{array}$$

If it require 110 yards of carpeting  $\frac{2}{3}$  of a yard wide, to carpet a room, how many yards,  $1\frac{2}{3}$  yards wide, will carpet the same?

$$\begin{array}{r|l} 11 & 8-2 \\ \hline & 43 \\ \hline & 110 \\ \hline & 60 \end{array}$$

The answer is 60 yards.

If I lend a friend \$100, for 40 days, how long ought he to lend me \$80, to return the accommodation?

$$\begin{array}{r|l} 2- & 80\ 100-5 \\ \hline & 40 \\ \hline & 50\ \text{da.} \end{array}$$

The answer must be in days.

How long must a board be, that is 9 inches wide, to make a square foot?

The question should be,

If 12 inches long and 12 inches wide, make 1 foot, what length, with 9 inches in width, will make the same?

Nine, the width, is evidently the demand, while 12 in width is the same name, and 12 in length is the term of answer; for the answer is required in length:

$\begin{array}{r} 12-9 \overline{) 12-4} \\ \underline{12-4} \\ 16 \text{ in.} \end{array}$	Without transferring and working the question anew, we may prove it by simply multiplying 9 and 16, which make 144, the number of square inches in a square foot.
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If a certain pasture supply 1500 cows 75 days, how long will it supply 2000 cows?

$\begin{array}{r} 2000 \overline{) 1500} \\ \underline{75} \\ 56\frac{1}{2} \end{array}$	The causes are, in this instance, active causes.
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If a man perform a journey in 24 days, when the days are 12 hours long, how many days will he be required to do the same, when the days are 18 hours long?

$\begin{array}{r} 2-18 \overline{) 12-2} \\ \underline{24-8} \\ 16 \end{array}$	The causes here, are all of the same kind, time; yet we know that the answer must be in days, and place the days, consequently, on the right.
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We will now present a few questions in the calculation of machinery; all of which, so far as *velocity* is concerned, are *inverse*.

If a drum, 60 inches in diameter, make 72 revolutions per minute, how many revolutions will a pulley connected with it, make, which is 6 inches in diameter?

It is necessary, in this question, to compare the two diameters; and, as 6 diameter is the demand, we place

it on the left, and 60 diameter on the right; while 72, the number of revolutions, becomes the denomination of the answer.

The answer is 720 revolutions

$$\begin{array}{r|l} 60 & 72 \\ \hline & 720 \end{array}$$

Let us now change this question, and find the diameter of this pinion, knowing the number of revolutions that we wish to make.

If 72 revolutions require 60 inches in diameter, of what diameter must a pinion be, to make 720 revolutions?

Thus, the diameter used at the outset is again found.

$$\begin{array}{r|l} 720 & 60 \\ \hline & 6 \text{ in.} \end{array}$$

Again: Making the two diameters of the demand thus found, the supposition, we will endeavor to find each term of the former supposition, 72 and 60.

If 6 inches in diameter make 720 revolutions, how many revolutions will 60 inches diameter make?

Here, the diameters must be compared.

$$\begin{array}{r|l} 60 & 6 \\ \hline & 720 \text{ rev.} \end{array}$$

If 720 revolutions require 6 inches diameter, what diameter will give 72 revolutions?

Thus, we can find the *diameter* and *revolution* of any wheel, the diameter and revolutions of that with which it is connected, being given.

$$\begin{array}{r|l} 720 & 6 \\ \hline & 60 \text{ in.} \end{array}$$

It frequently becomes necessary to ascertain the revolutions or diameter of a wheel connected with several others. This is done by what is called "conjoined proportion," which our limited space will not permit us to treat here.

Suppose a counter wheel in a mill turn 40 times per minute; how large must a trundle be on a spindle, to make 240 revolutions per minute, the counter wheel being 5 feet in diameter?

$\begin{array}{r} 240 \cancel{40} \\ - 60 \\ \hline 10 \text{ in.} \end{array}$	<p>Two hundred and forty revolutions is the demand, 40 revolutions the same name, and 5 feet, or 60 inches, the diameter of the counter wheel, is the term of answer. Hence, the trundle must be 10 inches in diameter.</p>
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How many inches in diameter must a water-wheel be, to make 12 revolutions, if a counter wheel, 60 inches in diameter, makes 40 revolutions per minute?

$\begin{array}{r} 12 \cancel{40} \\ - 60 - 5 \\ \hline 200 \text{ in.} \end{array}$	<p>These 200 inches make <math>16\frac{2}{3}</math> feet, which might be easily obtained by substituting 5 feet, for 60 inches, when the answer would be in feet. This shows us, that a water-wheel <math>16\frac{2}{3}</math> feet in diameter, making 12 revolutions per minute, working in a crown wheel of equal size, with a counter wheel, 60 inches in diameter, making 40 revolutions, must have a trundle 10 inches in diameter, to make 240 revolutions per minute. We may go on thus from one wheel to another, to any extent, and apply the diameter and revolutions of one to another, coming in regular sequence after it, until we ascertain the revolutions and diameters in a long chain of machinery.</p>
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Such calculations as the following, are frequently necessary in finding the size of wheels, their "pitch-line," and the number of "cogs," or teeth. They come properly under the head of *direct proportion*; but are placed in this connection, that most of the remarks of machinery may be found together. The problems are solved as other questions in simple proportion.

Suppose it be required to make a wheel  $1\frac{1}{2}$  inches larger, whose pitch-line is  $12\frac{1}{4}$  inches in diameter:

how many cogs will the wheel so enlarged have, if the first wheel had 67 cogs?

We first add  $1\frac{1}{2}$  to  $12\frac{1}{4}$  inches, making thus,  $12+1=13$  inches:  $\frac{1}{4}$  inch equals  $\frac{2}{8}$ , and  $8+15$  sixteenths equal  $\frac{23}{8}$ , or  $1\frac{7}{8}$ , which added to 13 makes  $14\frac{7}{8}$ , the diameter of the pitch-line, after adding the  $1\frac{1}{2}$  inches. We now say, as  $12\frac{1}{4}$  is to  $14\frac{7}{8}$ , so will 67 cogs be to the number required.

If a wheel  $12\frac{1}{4}$  inches in diameter, have 67 cogs, how many cogs will a wheel  $14\frac{7}{8}$  inches in diameter have; thus,

The demand is here placed on the right.

The wheel, thus increased in diameter, will have $74\frac{1}{2}$ cogs; and, as the number of cogs must always be even, we will suppose that the wheel shall have 75 cogs,	16 231
being the whole number that nearest expresses the fraction. Now, this will require the pitch-line to be still a little larger. So we say, if 67 cogs be advanced to 75 cogs, what will the pitch-line, $12\frac{1}{4}$ , be advanced to? Seventy-five cogs is the demand, 67 cogs, the same name, and $12\frac{1}{4}$ in diameter, the term of answer.	207 16
	67
	— 74 $\frac{1}{2}$

The answer is a very minute fraction under  $14\frac{1}{2}$  inches; and this pitch-line can be made so nearly the right diameter, as to step off with the "dividers" 75 cogs, without any perceivable fraction.

Suppose, again, a wheel, that is 18 inches in diameter, has 100 cogs; how many cogs must it have to be enlarged 6 inches?

Here,  $18+6=24$ , the diameter of the required wheel. Now, how many cogs will 24 the desired diameter require, if 18 inches, the given diameter, require 100 cogs; thus,

67 75	
16 207	
	— 14 $\frac{517}{1012}$

$\begin{array}{r} 18 \overline{) 24} \\ \underline{18} \\ 6 \end{array}$	<p>This gives for the number of cogs in the larger wheel <math>133\frac{1}{3}</math>, which not being an even number, we call 133, that from this smaller number of cogs, we may make the new pitch-line proportionally smaller.</p>
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If 100 cogs require 18 inches diameter, what pitch, diameter, will 133 cogs require?

$\begin{array}{r} 100 \overline{) 133} \\ \underline{100} \\ 33 \\ \underline{36} \\ 47 \end{array}$	<p>Therefore, to make 133 cogs, which is an even number, the new pitch-line must be <math>23\frac{4}{7}</math>, or 23.94 decimal inches; which, too, can be so accurately approximated, as to give 133, in about the desired diameter.</p>
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The article on Inverse Proportion, although short, has, we trust, a sufficient number and variety of examples to enable the judicious and discriminating reader to apply the principles in practice whenever found necessary.

We now close our remarks on the proportions; and shall hereafter advert to them only to show the philosophy of the statements that follow in the various rules of arithmetic and mensuration: as all of these subdivisions depend in principle and detail on the proportions; and are but different branches of them, assuming such *apparently* different form, as the circumstances of their application may require. We feel confident, that if the reader will carefully review the foregoing pages, with a close attention to principles, rather than practice, he will find no difficulty of perceiving the necessary operations to be performed on all questions that will follow in this work; nor in applying the principles that he has learned, in such manner as to conduct to satisfactory results. He cannot expect to do this, by a slight and indifferent course of reading: progress in arithmetical knowledge justifies no such conclusion, if we consult the history of those who have attained great proficiency in its principles. To become thoroughly acquainted with the science, requires not only

a due comprehension of each separate principle, but a knowledge of all their relative bearings; and even with these, such an acquaintance and familiarity, as to render their constant contemplation, which ever enables us to discern new beauties, rather a pleasure than a task. Nor must we regard cancelation as a short-hand system, merely, that will enable us to arrive at results *instantly*, without that thought necessary to the apprehension of great general principles in any science: for the abbreviation of this system, consists, not in the shortening of principles, but in their use and concentration. If, moreover, we investigate a proposition for the beauty and grandeur of its principles and relations, instead of as a forced and necessary task, for merely speculative purposes, we will not fail being thoroughly acquainted with it; thus connecting the pleasures of the science, with the necessity of the art.

From the foregoing illustrations, we deduce the following

#### SUMMARY OF DIRECTIONS FOR INVERSE PROPORTION.

*Ascertain first the term of Demand:*

*Place this Demand on the left:*

*Place the term of the same name of the demand, opposite the demand, on the right:*

*Place the term in which the answer is required, last on the right.*

*If any of the terms are fractional,*

*Place the Numerators on the side of the line ordinarily assigned to the integers, and the Denominator opposite*

## CONJOINED PROPORTION.

Conjoined Proportion differs from Simple Proportion, only by reason that, instead of one, *many* different proportions are found in one question. Hence, all questions coming under this head, may be wrought by continued separate statements in Simple Proportion. The *demand* is placed on the right, and the *same name*, in the supposition, on the left. This supposition merely equals in value the term in which the answer is required, and which is placed invariably on the right. Hence, statements in this species of proportion are always *direct*. Now, the term which is of the same name of that just used as denomination of answer, may be placed opposite this denomination of answer, and the new denomination of answer which this second term of supposition, or same name, equals, may be placed last on the right for a new term of answer. So, again, this term of answer may be used as demand, with another supposition opposite, and another term of answer succeeding, until any number of separate statements are merged into one general statement. Hence, the statement will be but a concatenation or chain of statements; from this fact, it has received the name of "*chain rule*." Nothing more is necessary, to make this interlinking of statements plain and easy, than to find the term of demand, or *that* term for which an equivalent term in value, is demanded. It will, therefore, be necessary, to place terms of the same denomination, and such only, opposite one another.

If 5 lbs. of raisins are worth 8 lbs. of tamarinds, and 3 lbs. of tamarinds are worth 18 lbs. of figs, and 20 lbs. of figs are worth 75 lbs. of cheese, and 40 lbs.

of cheese are worth 16 lbs. of butter, and  $18\frac{1}{2}$  lbs. butter are worth 30 yards of calico, and 5 yds. calico are worth  $4\frac{1}{2}$  bushels of apples, how many bushels of apples are worth  $13\frac{1}{2}$  lbs. of raisins?

It is evident, that the answer must be in bushels of apples, and that the demand is  $13\frac{1}{2}$  lbs. of raisins: hence, we conclude, that if raisins is the demand, raisins must be the same name; and consequently, place opposite the  $13\frac{1}{2}$  lbs. raisins, the term of raisins given in the question, which is 5; and so on with all of the other terms, making the 8 lbs. tamarinds, which this 5 lbs. equals, the term of answer, under the demand, and, again, the 3 lbs. of tamarinds, the same name, opposite, and so on; thus,

This may be proven by changing the conditions of the question. Again:

	4	5	5	—11
	5			
	3	5		
	20	15	—3	
5	—40	7	5	
	7	5	15	—8
			4	
		5	30	
			29	
		25	7128	
				285 $\frac{3}{4}$

If 3 days work of A, are equal to 5 of B, and 4 of B to 9 of C, and 10 of C to 6 of D, and 8 of D to 40 of E, and 3 of E to  $2\frac{1}{2}$  of F, and  $3\frac{1}{2}$  of F to  $18\frac{1}{2}$  of G, how many days' work of A will equal 25 of G?

Here, G is the demand, and certainly G must be the same name; hence, we state in the retrograde order of the question, until we find A, which was the first term proposed, and which is now the term of answer; thus,

5	—	7	5	2	5
				4	
				4	1
				5	2
				4	0
3	—	5	5	—	2
				5	1
				5	4
				5	
<hr/>					
		15	8		
<hr/>					
				15	of A.

The result is  $\frac{2}{15}$  of 1 day's work of A. The proportion in these questions is always direct; because the number on the left merely equals in value some other member on the right. Again:

If \$18 U. S. are worth 8 ducats, at Frankfort; 12 ducats at Frankfort, 9 pistoles, at Geneva; 50 pistoles at Geneva, 40 rupees, at Bombay; 16 rupees at Bombay, 20 rials, at Buenos Ayres; 15 rials at Buenos Ayres, 6 candarines, at Canton;  $4\frac{1}{2}$  candarines at Canton, 8 leptas, at Corinth; 2 leptas at Corinth, 5 aspers at Cairo; and 20 aspers at Cairo, 40 copecks, at St. Petersburg; how many copecks of St. Petersburg are worth \$120, U. States?

Dol.	18	120	Dollars.
Duc.	12	8	Ducats.
Pis.	50	9	Pistoles.
Rup.	16	40	Rupees.
Ri.	15	20	Rials.
Can.	9	6	Candarines.
		2	
Lep.	2	8	Lepta.
As.	20	5	Aspers.
		40	Copecks.
<hr/>			
		142	$\frac{2}{3}$

The answer is  $142\frac{2}{3}$  copecks.

All questions of a similar nature, in *exchange* and *reduction* of monies, may be wrought as the above, and without the least difficulty, when proper attention

is given to the foregoing, which may be brought into the following

#### SUMMARY OF DIRECTIONS FOR CONJOINED PROPORTION.

*Place the demand first on the right: Place the same name opposite; and place the term of answer, which equals the same name, under the demand, on the right. Again: Place opposite the term of answer, the term of the same name, and the term which equals the same name, under the former term of answer, on the right; and so on, until all of the terms are placed down. Cancel as in other cases.*

#### EQUATION OF PAYMENTS.

It frequently becomes necessary to make several different payments on a note at one time, or to make one payment of several different notes. This is done by *Equation of payments*, which simply means, to make the time of one payment equal to the average of the several periods. Equation is from the Latin, *equa*, equal. The following facts may be assumed:

\$ 1, in 3 months,	will gain as much as	\$ 3 in 1 mo
\$10, in 8 "	" " " "	" \$ 8 in 10 "
\$40, in 1 "	" " " "	" \$ 1 in 40 "
\$ 4, in 20 days	" " " "	" \$20 in 4 da.
\$17, in 11 "	" " " "	" \$11 in 17 "
\$ 8, in 1 year	" " " "	" \$ 1 in 8 yrs.
\$19, in 7 "	" " " "	" \$ 7 in 19 "

A merchant owes two notes, payable as follows: one for \$10, payable in 8 months, and the other for \$40, payable in 6 months. Now,

$$\begin{array}{rcl}
 \$10 \times 8 \text{ months equals } \$80, & \text{for 1 month.} & \\
 \$40 \times 6 & \text{“ “ } & \$240, \text{ for 1 “} \\
 \hline
 \$50 & & 50 \overline{) \$320}, \text{ for 1 “} \\
 & & \hline
 & & 6\frac{2}{3} \text{ months.}
 \end{array}$$

Above, each sum is multiplied by its number of months. The original sums are added, making \$50, for the equated time; or, their products in the months, making \$320, for 1 month. The latter product is divided by the former, giving the equated time  $6\frac{2}{3}$  months. The question would be stated thus, by Inverse Proportion:

If \$320 shall be paid in 1 month, in how many months must \$50 be paid? Thus,

$$\begin{array}{r|l}
 50 \overline{) 320} & \text{This is the proper method of arriving at} \\
 \hline
 1 & \text{the proportion in such questions. The an-} \\
 & \text{swer is, as before, } 6\frac{2}{3} \text{ months; because 1} \\
 6\frac{2}{3} & \text{month was the last term on the right.}
 \end{array}$$

Again: A owes to B 4 notes, payable as follows:

$$\begin{array}{rcl}
 \$25 \times \text{in 6 months} & = & \$150 \\
 \$75 \times \text{in 8 “} & = & \$600 \\
 \$200 \times \text{in 10 “} & = & \$2000 \\
 \$60 \times \text{in 3 “} & = & \$180 \\
 \hline
 \$360 & & \$2930
 \end{array}
 \left. \vphantom{\begin{array}{rcl} \$25 \times \text{in 6 months} \\ \$75 \times \text{in 8 “} \\ \$200 \times \text{in 10 “} \\ \$60 \times \text{in 3 “} \end{array}} \right\} \text{for 1 month.}$$

$$\begin{array}{r}
 360 \overline{) 2,930} \\
 \hline
 1
 \end{array}$$

Ans.  $8\frac{5}{8}$ , equal to 8 months, 4

days, and 4 hours.

I give 2 notes, one for \$100, payable in 6 months; the other for \$100, payable in 18 months: at what time may both be paid together?

$$\begin{array}{rcl}
 \$100 \times 6 \text{ months} & = & \$ 600 \\
 \$100 \times 18 \text{ " } & = & \$1800 \quad \left. \vphantom{\begin{array}{l} \$100 \times 6 \text{ months} \\ \$100 \times 18 \text{ " } \end{array}} \right\} \text{ for 1 month.} \\
 \hline
 \$200 & & 200 | 2400 - 12 \\
 & & \underline{1} \\
 & & 12 \text{ months, answer.}
 \end{array}$$

This method of calculating equation of payments, is not strictly correct; being based on the supposition that interest and discount are the same; that is, that the deduction made in advance, is equal to the interest that accrues, after a certain period. Thus, in the question above, \$100 are withheld from the creditor 6 months, and *interest* should be charged; whereas, another \$100 were paid 6 months before due, and should sustain only a *discount*. The interest on the money withheld, is greater than the discount on that advanced; hence the difference. This difference is, however, very minute, and of no practical importance.

When the time is days or years, such days or years must be multiplied into the sum to be paid, as in the cases of months above; and the equated time will be days or years, as the case may be. When the time is months and days, it must be reduced either to months or days.

If one of the debts is *paid down*, or a payment made at the date of the obligation, it will make no product in multiplying into time, and will be used only in finding the sum of the debts or payments.

From the foregoing, we deduce the following

#### DIRECTIONS FOR EQUATION OF PAYMENTS.

*Multiply the sum of each payment by its time. add the several payments: then, add their products; and divide the sum of the products, by the sum of the payments.*

*The several periods of time must be of the same denomination; either days, months, or years, separately; and the answer will be the equated time, in days, months, or years, as the case may be. Or, state as in Inverse Proportion.*

## FELLOWSHIP, SIMPLE AND COMPOUND.

INCLUDING PARTNERSHIP, GENERAL AVERAGE, BANKRUPTCIES, ETC.

In Simple Fellowship or Partnership, General Average, Bankruptcy, etc., two or more individuals, on different sums of money, gain or lose some general sum; when, it is desired to know each individual gain or loss.

In Compound Fellowship, not only the sums of capital are different, but are invested for different periods of time, etc. From this fact it takes the name of Compound Fellowship.

A, B, and C invest \$1000 in a cargo of wheat: A puts in \$200, B \$300, and C \$500; and agree to share the profits and losses, in proportion to the capital severally invested. They gain \$600; what is each man's share?

A's share	\$200
B's    "	300
C's    "	500

\$1000. Sum of gain \$600.

Now, the whole sum, 1000, gains the whole sum, 600; therefore, we may ask, what share of 600 each

individual share gains. The statements, according to proportion, are as follows:

If 1000 gain 600, what will 200 gain?  
 " " " " " " 300 "  
 " " " " " " 500 "

$$\begin{array}{r} \cancel{1000} \overline{) 200} \\ \underline{600} \\ \$120 \end{array}$$

$$\begin{array}{r} \cancel{1000} \overline{) 300} \\ \underline{600} \\ \$180 \end{array}$$

$$\begin{array}{r} \cancel{1000} \overline{) 500} \\ \underline{600} \\ \$300 \end{array}$$

A's share of gain 120  
 B's " " " 180  
 C's " " " 300

Whole sum gained,  $\$600$  Proof.

A father divides  $\$1800$  among three sons, in the following proportion: A, 1 share; B, twice as much as A; and C, three times as much as B; what is the share of each?

Let us set A's share down as a unit; thus, 1  
 B's share, as twice this, 2  
 C's share, as three times the latter, 6  
 Sum of the shares, 9

Thus, there are 9 shares; and A has  $\frac{1}{9}$ ; B,  $\frac{2}{9}$ , and C,  $\frac{6}{9}$ : hence, their shares would be as 9 to 1; 9 to 2; and 9 to 6; thus,

$$\begin{array}{r} \$1800-2 \\ \underline{1} \\ 200 \end{array}$$

$$\begin{array}{r} \$1800-2 \\ \underline{2} \\ 400 \end{array}$$

$$\begin{array}{r} \$1800-2 \\ \underline{6} \\ 1200 \end{array}$$

Or, the questions might be stated, thus,

What will 1 share be, if 9 shares be 1800? What will 2 shares be? What will 6 shares be, etc.?  $200 + 400 + 1200 = \$1800$ , the estate; which proves the work correct.

A owes to B \$400;  
 " " " C \$600;  
 " " " D \$500;  
 " " " E \$800;  
 " " " F \$200; and pays only \$2500  
 much does each lose? \$2500 \$900

The whole sum of the credits loses \$900: hence, the following statements:

$$\begin{array}{r}
 2500 \overline{) 400} \\
 \underline{900} \phantom{00} 4 \\
 \$144 \text{ B.}
 \end{array}
 \quad
 \begin{array}{r}
 2500 \overline{) 600} \\
 \underline{900} \phantom{00} 4 \\
 \$216 \text{ C.}
 \end{array}
 \quad
 \begin{array}{r}
 2500 \overline{) 500} \\
 \underline{900} \phantom{00} 4 \\
 \$180 \text{ D.}
 \end{array}$$

$$\begin{array}{r}
 2500 \overline{) 800} \\
 \underline{900} \phantom{00} 4 \\
 \$288 \text{ E.}
 \end{array}
 \quad
 \begin{array}{r}
 2500 \overline{) 200} \\
 \underline{900} \phantom{00} 4 \\
 \$72 \text{ F.}
 \end{array}$$

$144 + 216 + 180 + 288 + 72 = 900$  dollars, the whole sum lost.

A man left to his four sons \$60, to be divided in the proportion of  $\frac{1}{8}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ; how much does each one get?

$$\begin{array}{l}
 \frac{1}{8} \text{ of } 60 = 20 \\
 \frac{1}{4} \text{ " } 60 = 15 \\
 \frac{1}{5} \text{ " } 60 = 12 \\
 \frac{1}{6} \text{ " } 60 = 10
 \end{array}$$

Then, as  $57 : 60 :: 20 : X$  or  $21\frac{3}{7}$   
 " "  $57 : 60 :: 15 : X$  or  $15\frac{1}{2}$   
 " "  $57 : 60 :: 12 : X$  or  $12\frac{2}{3}$   
 " "  $57 : 60 :: 10 : X$  or  $10\frac{2}{3}$   
 \$60

$$\begin{array}{r}
 57 \overline{) 20} \\
 \underline{60} \\
 21\frac{3}{7}
 \end{array}
 \quad
 \begin{array}{r}
 57 \overline{) 15} \\
 \underline{60} \\
 15\frac{1}{2}
 \end{array}
 \quad
 \begin{array}{r}
 57 \overline{) 12} \\
 \underline{60} \\
 12\frac{2}{3}
 \end{array}
 \quad
 \begin{array}{r}
 57 \overline{) 10} \\
 \underline{60} \\
 10\frac{2}{3}
 \end{array}$$

The several sums, added, make the original \$60.

The \$57 was the deficient sum of capital; and 20, 15, 12, and 10, the deficient individual sums. Then, if 57, deficient, be advanced to 60, full sum, what will any of the other deficient sums be advanced to, for full sum or share?

Railroad, canal, bank, and other stock *dividends*, or *assessments*, may be calculated as the first and second examples given.

#### GENERAL AVERAGE. -

It frequently becomes necessary, at sea, to throw overboard a large portion of the cargo, to secure safety in time of storm. The property thus sacrificed may belong to one individual, although it is thus thrown overboard for the benefit of the whole. Hence, the whole cargo should sustain a loss proportionate to the value of each individual interest. Now, it becomes necessary to assess the loss according to value: this is called *General Average*; while the property ejected is called *jettison*, from the French, *jetter*, to throw. All such losses are sustained by the *ship*, the *cargo*, and the *freight*, according to the value of each, which is called *pro rata*, or *in proportion*.

Ordinary losses, such as *wear* and damage, or sacrifice made for the safety of the ship alone, must be borne by the owners of the vessel; losses made for the safety of any particular portion of the cargo, must be charged to the individual so losing, and are not to be brought into the general average.

The property lost must be reckoned, as well as that saved.

The cargo is valued at the price it would bring at the place of destination, after deducting storage and all necessary charges.

One-third is generally deducted from the freight, for

wages, pilotage, etc., etc.; in New York, one-half is allowed.

One-third of the cost of repairs, on masts, spars, rigging, etc., is deducted before the average is made; thus making the valuation of the old two-thirds of the new. This is done on the principle of insuring only two-thirds of the value of property on land; or to allow for damage. And here, the danger must be imminent, or the general average will not be allowed.

All necessary charges must be deducted from each individual's interest before the average is made.  
Example:

The ship, John Adams, from Havre to Boston, had on board a cargo estimated at \$60,000. Of this A owned \$20,000; B, \$30,000; and C, \$10,000. The gross amount of freight and passage money was \$12,000. The ship was worth \$50,000; and \$800 had been paid for insurance. The ship, being in great distress, the master threw overboard \$7,800 worth of goods, and cut away her masts, rigging, and anchors. In port, it cost \$3,000 for repairs; what was the loss of each owner, both of ship and cargo.

Ship valued at . . . . .	\$50,000	
Premium deducted, . . . . .	800	\$49,200
Cargo worth, . . . . .		60,000
Freight and passage, . . . . .	12,000	
One-third deducted for wages, . . . . .	4,000	8,000
Sum of individual interests, . . . . .		\$117,200
Goods thrown overboard, . . . . .		7,800
Cost of new masts, spars, etc., . . . . .	\$3,000	
One-third deducted for wear, . . . . .	1,000	2,000
Com. on repairs, . . . . .		20
Port duties and other expenses, . . . . .		186
Sum of loss, . . . . .		\$10,000

As	117,200	:	20,000	::	10,000	:	\$1706.48	A's loss
As	"	:	30,000	::	"	:	2559.73	B's "
As	"	:	10,000	::	"	:	853.24	C's "
As	"	:	49,200	::	"	:	4197.95	Ship's l.
As	"	:	8,000	::	"	:	682.60	Frt's l.

PROOF. Whole loss,  $\$10,000.00$  as ab've.

## COMPOUND FELLOWSHIP.

A, B, and C purchase a pasture to be used by them jointly, for \$50, in which A keeps 80 oxen 3 months; B, 100 oxen 2 months; and C, 160 oxen 1 month; what part of the cost must each pay?

It is not necessary here that the time be expressed as months; for it is simply a unit; and 3, 2, and 1 show the ratios of consumption of grass, rather than the time; as the \$50 pay alike for 1 month, 1 year, or 10 years. The multiplication of the time and oxen together, shows merely how many oxen, in each case, remain a common length of time in the pasture; for, after being thus multiplied, one lot is supposed to be in the pasture as long as the other; and the difference of consumption and price is the effect of the different numbers of oxen thus grazing.

A's  $80 \times 3$  months = 240 oxen for the time.

B's  $100 \times 2$  " = 200 " " " "

C's  $160 \times 1$  " = 160 " " " "

The whole = 600 " " " "

Now, if 600 oxen cost \$50, what will 240, 200, and 160 cost, respectively; thus,

$$\begin{array}{r} 600 \overline{) 240} \\ \underline{50} \\ 20 \text{ A's.} \end{array}$$

$$\begin{array}{r} 600 \overline{) 200} \\ \underline{50} \\ 16\frac{2}{3} \text{ B's.} \end{array}$$

$$\begin{array}{r} 600 \overline{) 160} \\ \underline{50} \\ 13\frac{1}{3} \text{ C's.} \end{array}$$

PROOF.  $20 + 16\frac{2}{3} + 13\frac{1}{3} = \$50$

A and B companied; A put in \$2,000, January 1; but B put in June 1; what sum did he put in, to have an equal share of the profits with A?

A has 2,000 employed 12 months; it is, therefore, desired to know how much capital belonging to B will, in 7 months, make equal profits with the 2,000, 12 months. Hence, the question is one of inverse proportion, and is wrought thus,

$$\begin{array}{r|l} 7 \overline{) 12} & \\ \hline 2000 & \text{The demand is on the left.} \\ \hline \$34284 & \end{array}$$

"In an adventure, A put in \$12,000 for 4 months; then adding \$8,000, he continued the whole 2 months; B put in \$25,000, and after 3 months took out \$10,000, and continued the rest 3 months longer; C put in \$35,000 for 2 months; then, withdrawing  $\frac{2}{3}$  of his stock, continued the remainder 4 months longer: they gained \$6,000; what was the share of each?

$$\begin{array}{l} \text{A's } \$12,000 \times 4 \text{ mos.} = 48,000, \\ \text{A's } \$20,000 \times 2 \text{ " } = 40,000, \end{array} \left. \vphantom{\begin{array}{l} \text{A's } \$12,000 \times 4 \text{ mos.} = 48,000, \\ \text{A's } \$20,000 \times 2 \text{ " } = 40,000, \end{array}} \right\} = 88,000.$$

$$\begin{array}{l} \text{B's } \$25,000 \times 3 \text{ " } = 75,000, \\ \text{B's } \$15,000 \times 3 \text{ " } = 45,000, \end{array} \left. \vphantom{\begin{array}{l} \text{B's } \$25,000 \times 3 \text{ " } = 75,000, \\ \text{B's } \$15,000 \times 3 \text{ " } = 45,000, \end{array}} \right\} = 120,000.$$

$$\begin{array}{l} \text{C's } \$35,000 \times 2 \text{ " } = 70,000, \\ \text{C's } \$25,000 \times 4 \text{ " } = 100,000, \end{array} \left. \vphantom{\begin{array}{l} \text{C's } \$35,000 \times 2 \text{ " } = 70,000, \\ \text{C's } \$25,000 \times 4 \text{ " } = 100,000, \end{array}} \right\} = 170,000.$$

$$\text{Whole sum,} \quad \$378,000$$

If 378,000 gain 6,000, what will 88,000, A's, gain?

If 378,000 " " " " 120,000, B's, "

If 378,000 " " " " 170,000, C's, "

$$\begin{array}{l} \text{A gains } \$1,396.83, \\ \text{B " } \$1,904.76, \\ \text{C " } \$2,698.41, \end{array} \left. \vphantom{\begin{array}{l} \text{A gains } \$1,396.83, \\ \text{B " } \$1,904.76, \\ \text{C " } \$2,698.41, \end{array}} \right\} = 6,000, \text{ PROOF.}$$

From the examples given, we deduce the following

**SUMMARY OF DIRECTIONS FOR SIMPLE FELLOWSHIP.**

*Add the several sums: make the whole sum the supposition; each separate sum, the demand; and the whole gain or loss, the term of answer: the answer will, in each case, be the gain or loss on the individual sums.*

**PROOF:** *Add the several answers, and the whole sum will equal the whole gain or loss.*

*To make General Average,*

**FIRST:** *Ascertain the value of the cargo; the sum of the freight-bill, passage, etc., minus one-half, or one-third, as customary: the sum total will be the general value:*

**SECOND:** *Ascertain the sum of loss, goods thrown overboard; cost of new masts, spars, rigging, etc., minus one-third; commission on repairs; port duties, and other expenses: add these, and the sum total will be the loss. Then, make each individual loss the demand; the sum of all the combined interests, the same name; and the whole sum of loss, the term of answer.*

**PROOF:** *The several answers, added, will equal the whole loss.*

**COMPOUND FELLOWSHIP.**

*Multiply each sum of capital by the time invested; add the products; make each separate product the demand; the sum of the products, the same name; and the whole gain or loss, the term of answer.*

**PROOF:** *Add the several specific sums, and they will equal the whole sum of gain or loss.*

*When a part of an investment is deducted, or an additional sum paid in, make the remainder, or*

*the increased sum, as the case may be, a new investment for the time that it runs; and multiply by the time, as before.*

*When two or more investments, made by one individual, are multiplied thus, the separate products may be added into one individual product, before being placed on the line.*

## BARTER, DUTIES, AND COMMERCIAL EXCHANGE.

Barter\* is the exchange of one article of specified value, for an equivalent value in something else. Hence, articles in Barter, or Commercial Exchange, are considered in the ratio of their separate *values* or *quantities*.

How many bushels of wheat at 50 cents per bushel, will pay for 200 bushels of corn, at 30 cents per bushel?

The demand here, is, what will  $200 \times 30$  cents buy, if 50 cents, opposite, buy 1 bushel? Thus

$$\begin{array}{r} \$\phi\phi\phi\phi-12 \\ \hline 1 \\ \hline 120 \text{ bu.} \end{array}$$

The answer is 120 bushels of wheat. The 30 and 200 are here multiplied, merely to show that the demand is the price of the whole of the corn; otherwise the proportion would not be recognized. It should be stated with the terms to be multiplied, one above the other; thus,

\* Barter is from the Spanish, *baratar*, from the Latin root, *certo*, to turn or exchange. This change implies equality in the articles, or the prices of the articles, exchanged. From its derivation, it bears a striking resemblance to the word *proportion*.

One hundred and twenty, as before.

$$\begin{array}{r} 200 \\ 50 \overline{) 30} \\ \hline 120 \end{array}$$

How many bushels of corn, at 30 cents per bushel, will pay for 120 bushels of wheat, at 50 cents per bushel?

The answer is 200 bushels of corn.

$$\begin{array}{r} 20 \overline{) 400} \\ 40 \overline{) 200} \\ \hline 200 \end{array}$$

If 200 bushels of corn, at 30 cents per bushel, pay for 120 bushels of wheat, what is the price of the wheat?

The answer is 50 cents.

$$\begin{array}{r} 2 \overline{) 100} \\ 20 \overline{) 50} \\ \hline 50 \text{ cts.} \end{array}$$

How many pounds of butter, at  $12\frac{1}{2}$  cents per lb., will pay for  $18\frac{1}{2}$  lbs. of bacon, at 10 cents per lb.?

Here, the demand is  $18\frac{1}{2}$  times 10 cents; the same name  $12\frac{1}{2}$  cents; and the term of answer, 1 pound of butter: hence, 15 lbs. of butter, the answer.

$$\begin{array}{r} 2 \overline{) 185} \\ 20 \overline{) 100} \\ \hline 15 \text{ lbs.} \end{array}$$

How many pounds of sugar, at  $3\frac{1}{2}$  cents per lb., will pay for 20 bbls. of rum, 35 galls. to the barrel, worth 90 cents per gallon?

The answer is 18,000 pounds of sugar.

$$\begin{array}{r} 20 \\ 2 \overline{) 40} \\ 35 \overline{) 100} \\ \hline 18,000 \end{array}$$

If 18,000 lbs. of sugar pay for 20 bbls. of rum

35 gallons to the barrel, and worth 90 cents per gallon, what is the sugar worth per lb.?

$$\begin{array}{r|l}
 2-18000 & 1 \\
 & 20 \\
 & 35-7 \\
 & 90 \\
 \hline
 & 3\frac{1}{2}
 \end{array}$$

What will 1 lb. of sugar cost, if 18,000 lbs. cost  $20 \times 35 \times 90$  cents?

If 18,000 lbs. sugar, at  $3\frac{1}{2}$  cts. per lb., pay for 20 bbls. of rum, worth 90 cents per gallon, how many gallons are there per barrel?

$$\begin{array}{r|l}
 20 & 18000-5 \\
 & 27 \\
 & 90 \\
 \hline
 & 35 \text{ galls.}
 \end{array}$$

It is seen in this and the preceding examples, that all of the constituents of the given commodity are placed on the right, and the remaining constituents, of the commodity about which

the inquiry is made, on the left. If there are four constituents on the right for the perfect supposition, and three out of four given, for an equivalent in barter, the three given, must be placed on the left, and the other factor or constituent will be found. It is observed that *the price is placed on the left, to find its associated quantity; or, the quantity is placed on the left, to find its associated price.*

#### BARTER BY REDUCTION

All reduction, ascending and descending, is a species of concatenated or conjoined proportion. Where articles are of different denominations, or are paid for in prices of different denominations, it is very convenient to make these reductions on the line, at the same time that the general question is wrought. And, if treated proportionally, the statement becomes lucid, and the work interesting.

How many yards of cloth, at \$1,25 per yard, will pay for 6 tons of iron, and 5 pence per lb., in the cur-

rency\* of New England, which is 6 shillings to the dollar?

What will 6 tons make, if 1 ton make 20 cwt.; what will this make, if 1 cwt. make 100 lbs.; what will all of these lbs. come to, if 1 lb. cost 5 pence; how many shillings will these pence make, if 12 pence, opposite, make 1 shilling; how many cents will all of these shillings make, if 6 shillings, New England, opposite, make 100 cents; how many yards will all these cents buy, if 125 cents buy 1 yard? Hence, the answer is in yards,  $666\frac{2}{3}$ ; the number that will pay for the 6 tons.

Let us now prove it, by asking, how many tons will pay for  $666\frac{2}{3}$  yards. The latter number becomes the demand; thus,

Ton.	16	Tons.
Cwt.	120	Cwt.
Lb.	1100	Lbs.
D.	125	Pence.
Sh.	61	Shil.
Cts.	125	100 Cents.
	1	Yard.
<hr/>		
Ans. $666\frac{2}{3}$ yds.		

The ones are wholly unnecessary in the solution; and are given merely to indicate the ratio

	3	
Yd.	1	2000 Yds.
Cts.	100	125 Cts.
Sh.	16	Sh.
D.	5	12 D.
Lbs.	100	1 Lb.
Cwt.	20	1 Cwt.
	1	1 Ton.
<hr/>		
Ans. 6 tons.		

\* On the adoption of Federal Money by the United States, it was found that the paper currency, issued by the bank of England, for her colonies in America, had in all of the colonies depreciated in value; in some, more, and in others, less, according to the embarrassment of the colony. Hence, when it became necessary to represent this currency in each state by our specie standard, it was found that in New England and Virginia, 6 shilling bills were worth 1 dollar; in New York and North Carolina, 8; in Pennsylvania, New Jersey, Delaware, and Maryland, 7s. 6 d.; and in Georgia and South Carolina, 4s. 8 d. The New England currency has been adopted by Kentucky, Tennessee, Illinois, Indiana, Missouri, and Mississippi; and that of New York, by Michigan and Ohio. Hence, the reason why the currency is different in different states. These methods of reckoning money are now very little used, and should be entirely superseded by our beautiful decimal system. Sterling money derived its name from *Estor-Nag*, who first made the coin. The dollar mark is a combination of U. S. and was originally written U. S. 20.00, etc.

We may prove it, again, by finding the cost of 1 lb. of iron, which was 5 d.; hence, the 5 d. is the demand:

D.	125	D.
Sh.	61	Sh.
Cts.	125100	Cts.
Yds.	20001	Yd.
	3	3
Ton.	16	Tons.
	120	Cwt.
	100	Lbs.

*Ans.* 1 lb. iron.

This may be proven, by demanding cost of 1 lb. iron:

Lbs.	1001	Lb.
Cwt.	201	Cwt.
Tons.	61	Ton.
	3	
Yd.	12000	Yds.
Cts.	100125	Cts.
Sh.	16	Sh.
	12	D.

*Ans.* 5 pence.

Denominations of the same kind are placed opposite each other.

This might be proven by several other processes; but further proof is unnecessary.

How many galls. cordial at \$180 per gal., will pay for 10 T. wine, at 6 d. per pt., S. Carolina currency?

1	10
12	
12	
162	—3
14	
12	
126	
201	
71	
1.30	30.00
1	

*Ans.* 1200 gallons.

Tun.	110	Tuns.
Pipe.	12	P.
Hhd.	12	Hhds.
Gal.	163	Gals.
Qt.	14	Qts.
Pt.	12	Pts.
D.	126	D.
Sh.	201	Sh.
£.	71	£.
Cts.	1.8030.00	Cts.
	1	Gal.

*Ans.* 1200 gallons.

The £7, above equal \$30, or 3,000 cents, South Carolina currency.

In this question, the demand is 10 tuns; the same name, 1 tun: and the term of answer, the next lower denomination, 2 pipes. Thus, *the answer of every preceding statement, is made the demand of a subsequent question.*

This question may be proven in several ways, as the one preceding.

How many barrels of wheat, at 50 cts. per bushel, will pay for 12 tons of iron, at 4 pence per lb., New York currency?

1	12
1	20
1	100
12	4
8	1
50	100—4
5	1
1	
<hr/>	
Ans.	400

Ton.	1	12	Tons.
Cwt.	1	20	Cwt.
Lb.	1	100	Lbs.
D.	12	4	D.
Sh.	8	1	Sh.
Cts.	50	100	Cts.
Bu.	5	1	Bush.
		1	Barrel.
<hr/>			
Ans.	400	barrels.	

In New York, 8 shillings make 1 dollar; hence, the 8 on the left of the line.

From the foregoing, we deduce the following

#### DIRECTIONS FOR BARTER, COMMERCIAL EXCHANGE, ETC.

*Place all of the terms of the commodity whose quantity and value are given, on the right; and all of the terms of the commodity whose quantity or value is required, on the left: the answer will be the quantity or value, as the case may be.*

**FOR DENOMINATE NUMBERS:** *Place the demand first on the right: place a unit of the same name, or the quantity specified of the same name, opposite: place the number which this unit makes in reduction,*

*or such other quantity of something else as the given quantity may equal, on the right: place, again, opposite this last term, on the left, the unit or other quantity of the same name, and proceed as before; making the answer of each preceding, the demand of a succeeding question, ad infinitum.*

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### DUTIES, TARE AND TRET, ETC.

We shall bring together and consider in one article, both Duties and Tare and Tret, because of their almost entire identity, and the similarity of operations necessary to reduce them; both depending alike on per centage, as well as losses of *tare, tret, leakage, etc.*

Commercial duties are sums of money paid to a government, for the privilege of importing foreign goods. The general law imposing these duties, and determining their rate and extent, is called a *tariff*. Tariffs are different in different countries, varying in their extent according to the necessities which cause their creation. In some countries, a charge is made on goods, produce, etc., for the ostensible purpose of creating revenue to discharge the expenses of government; in others, for the purpose of protecting manufactures, or agriculture, by preventing the importation of foreign manufactures or agricultural products. The manufacturing facilities of Great Britain being such that she can, not only compete with, but defy the whole world, she lays a tariff more particularly on the products of the soil, both for the promotion of industry, with the full development of her natural resources, and the establishment, among

Tret is from the Latin, *tritus*, from *tero*, to wear; Tare is French, and is from the same root as the Italian *tarare*, to abate.

her laboring population, of a long-cherished system of almost arbitrary tenantry.

In our own country, to the contrary, a tariff is imposed for the avowed purpose of making the premium on foreign privileges, pay the expenses of our government. Such a tariff, varies in its exactions, according to the financial exigencies of the government.

We have had a tariff in our country for quite a different purpose; that of fostering her domestic manufactures. This was necessarily high enough to prevent a large foreign importation, by rendering the profits accruing, too small for consideration.

Our federal constitution expressly forbids any tariff between the several states of this union.

1. In every port in the United States, where vessels are permitted to enter, a *customhouse* is established by the government, at which the duties assessed on goods, are paid. The officer who examines the cargo, determines the duties, etc., is called a *custom-house officer*. All importations, whether by our own, or foreign vessels, are alike subject to these duties.

2. Duties are divided into *specific* and *ad valorem*.

3. A *specific* duty is a certain sum charged on some specified quantity, as a cwt., gallon, yard, ton, foot, etc. This kind of duty is levied without reference to price.

4. An *ad valorem* duty is a certain per cent. on the price of the article. This phrase is from the Latin, *ad*, according to, and *valorem*, value; according to value.

5. When specified duties are received, certain allowances are made for *tare*, *tret* or *draft*, *leakage*, etc. These allowances are sometimes a specified per cent. on the original quantity; and sometimes a specified deduction on the cwt., cask, box, etc.; and are deducted before the duty is received; otherwise duty would be reckoned, alike on the quantity wasted; and that saved.

6. *Tare* is an allowance made for the box, bag, crate, etc., which contains the article; and is deducted, either specifically or at a certain per cent. It has been customary to allow 12 lbs. on the 112, or old cwt.

7. *Tret* is a certain allowance on the weight after tare is deducted, for waste, grease, dust, and other extraneous substances, and is usually 4 lbs. on the 104.

8. *Leakage* is an allowance, usually of 2 per cent., on liquids, such as liquors, oils, chemicals, etc., for waste.

9. *Gross weight* is the weight before any deductions are made.

10. *Nettle* is the weight after a part of the deductions are made; as the weight, after deducting tare, from which tret is to be deducted.

11. *Net weight* is what remains, after all deductions are made; and is the basis on which duties are reckoned.

12. The rates of loss are different on different articles, according to the law regulating such deductions. Losses of this kind are sometimes allowed in individual transactions, in groceries, etc.

### SPECIFIC DUTIES.

What is the specific duty on 1900 gallons of molasses, at 9 cents per gallon, allowing 2 per cent. for leakage?

The first thing to be done, is to find the number of gallons, after deducting the leakage. What will 1900 be reduced to, if 100 be reduced to 98; thus,

1900	1900	Here, 1900 gross is the demand, and 100 gross, the same name; while 98, net, is the term of answer. We multiply the net number of gallons by 9 cents, the specific duty, and find the duty, \$167,58, cutting off two figures, for cents. We should have placed 1 opposite, and 9 under the 98, saying, what will all of the net come to, if 1 gallon, net, be 9 cents?
	98	
	1862	
	9	
	\$167.58	

What is the duty on 20 hhds. whisky, at 10 cents per gallon, allowing 2 per cent. for leakage?

120	The hhds. are reduced to gallons, as in other cases
100	
198	
10	
\$123.48	

At 6 cents per lb., what is the specific duty on 170 kegs of tobacco, weighing each 125 lbs, allowing 6 lbs. per hundred for tare?

When tare is so much per box, bag, etc., we multiply by the number of boxes, bags, or whatever they be: find the whole tare, and after subtracting this from the whole gross, multiply the net remainder by the specific rate; thus,

$$\begin{array}{r|l}
 117\phi & \\
 \hline
 \$100 & 125-25 \\
 & 194 \\
 \hline
 & \phi-3 \\
 \hline
 & \$1198.50
 \end{array}$$

What is the duty, at  $4\frac{1}{2}$  cents per lb., on 80 boxes tobacco, weighing each 100 lbs., allowing tare at 20 cents per box?

Here, the tare would be, at 20 lbs. per box, on 80 boxes,  $20 \times 80 = 1600$  lbs. Now,  $8000 - 1600 = 6400$  lbs., net. This net weight is multiplied by the specific rate; thus,

We find that the duty is \$288.00. Thus, it is seen, that we can use the line to advantage, even in specific duties.

$$\begin{array}{r|l}
 6400-32 & \\
 \hline
 29 & \\
 \hline
 & \$288.00
 \end{array}$$

## DIRECTIONS FOR SPECIFIC DUTIES.

*Find the net weight, and multiply this by the duty.*

*To find the net weight, make the gross sum the demand: 100 gross the same name: and 100, diminished by the tare, tret, or leakage, the term of answer: the answer will be the net weight. Place under the term representing net weight, the duty; cut off two figures in the answer, when the duty is in cents, and the answer will be in dollars and cents.*

## AD VALOREM DUTIES.

It may be again observed, that ad valorem duties are reckoned on the cost price of the article; hence, no deductions of tare, tret, etc., are made, as in specific duties. The duty is reckoned on the invoice of goods, in the same manner that premium is reckoned on insurance.

1. An *invoice* is a bill giving in all the goods, and setting forth the price of each article.

2. To prevent deception, this bill must be *verified*, or sworn to, by the owners, or one of the owners of the goods, to the effect, that the invoice presents a true statement of cost prices; and that no *discount*, *drawback*, or *bounty* has been named, that has not been actually allowed.

3. This oath shall be administered by a *consul*, commercial agent, or other duly authorized officer, in the country where the goods are purchased; which fact shall be duly certified by such consul or agent.

4. The officer thus acting, who commits any fraud, in making the invoice too small, etc., is liable to heavy penalties. Our government keeps a consul, or other commercial agent, in every important part of the world, for this purpose, except where they are forbidden admittance by the laws of the country, as has, until recently, been the case in China.—*See Laws of the United States.*

What is the ad valorem duty, at 20 per cent., on a lot of boots and shoes, worth \$40,000?

The question is, what duty will \$40,000 give, if \$100 give \$20 duty? thus,

$$\begin{array}{r|l} 100 & 40,000 \\ \hline & 20 \\ \hline & \$8,000 \end{array}$$

We find that the duty is \$8,000.

A lot of Italian silks cost \$9,100; what is the ad valorem duty on them, at 43 per cent.? thus,

$$\begin{array}{r|l} 100 & 9,100 \\ \hline & 43 \\ \hline & \$3,913 \end{array}$$

The answer is \$3,913.

What is the duty on an invoice of socks, which cost \$3,963, at 25 per cent.? thus,

$$\begin{array}{r|l} 4-100 & 3963 \\ \hline & 25 \\ \hline & 990\frac{3}{4} \end{array}$$

We might suspend the 100 on the left, and cut off two figures for decimals of a dollar, on the right.

When the rate is the aliquot part of a dollar, we may very much abridge the operation by suspending the 100 on the left, and multiplying by such fractional part of a dollar, as the rate makes, as above: 25 per cent. is the  $\frac{1}{4}$  of 100; hence, we may multiply the 3963 by  $\frac{1}{4}$ , which is equivalent to simply dividing it by 4.

At 22 per cent., what is the duty on \$71,000 worth of crockery-ware?

This answer is obtained in dollars.

100	71,000
	22
	—
	\$15,620

What is the duty on 20 tons of wool, at 80 cents per lb., the duty being 60 per cent.?

The first point is to know how much the wool comes to in cents; hence, the reduction descending, found in the statement. After being reduced to pounds, it is multiplied by the price per lb.; 100 is placed opposite; and the rate, last on the right.

1	20
1	20
1	100
100	30
	60
	—
	\$7200.00

We will now consider *tare* and *tret* in a different form. In ordinary cases coming under this head, 12 lbs., on the 112, are deducted for tare. This 112 lbs. was the old standard for 1 *cwt.*; but is not now used. It has been customary to deduct 4 lbs. per 104 for tret, after the deduction of the tare. This custom has pretty much gone out of use. The 12 lbs. deducted for tare, should be deducted from 112; not from 100, as is dishonestly done by some dealers. Deducting 12 from the 100, instead of 112, is similar in principle to the deduction of false discount.

In this, as in many of the departments of arithmetic, too much attention is given to questions merely theoretical, in which denominate numbers are used, such as have no existence in business. We never hear anything of *lbs.*, *grs.*, *cwt.*, etc., in business. Hence, the impropriety of introducing such questions into a practical treatise.

It is to be regretted that our whole system of denominate numbers, with the exception of Federal Money and Avoirdupois Weights, is so varied and irregular; while the decimal substitute could be made with great ease and palpable advantage.

We subjoin a few questions involving commercial tare and tret:

What will 1 ton, gross weight, of hemp come to, at  $\$3,62\frac{1}{2}$  cents per gross cwt.?

$\begin{array}{r} 112 \overline{) 2240} \\ 2725 \\ \hline 72,50 \end{array}$	We here use 2240 lbs. as the gross ton. The price per cwt., or per 112 lbs., gross, being in cents, the answer is cents.
--	--

Or, what will 20 cwt. cost, if 1 cwt. cost  $\$2,62\frac{1}{2}$ ? thus,

$\begin{array}{r} 1 \overline{) 20} \\ 725 \\ \hline 72,50 \end{array}$	The answer is $\$72,50$ , as before.
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What will 8 tons tobacco come to, at 10 cents per lb., allowing 10 lbs. per 100 for tare?

$\begin{array}{r} 1 \overline{) 8} \\ 120 \\ 190 \\ 10 \\ \hline 1440,00 \end{array}$	The answer is $\$1440,00$ .
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What would be the duty on the same at  $37\frac{1}{2}$  per cent.?

$\begin{array}{r} 1 \overline{) 8} \\ 120 \\ 190 \\ 100 \\ 275 \\ \hline 540,00 \end{array}$	Above, 1 ton equals 20 cwt., and 1 cwt. equals 90 lbs., net; while 1 lb. net equals 10 cents, and 100 cts., give $\frac{1}{2}$ cents duty.
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I have 1500 lbs. of cotton, worth 5 d. sterling per lb., net; and wish to know the duty, if I allow 15 lbs. per 100 for tare, and 4 lbs. per 100 for tret, the duty being 40 per cent.?

We reduce the 5 d. to Federal money, by using £1

on the left, which equals \$4.84 on the right; or by saying that 20 shillings equal \$4.84.

This combination is quite simple, and enables us to arrive at the result with very few figures.

	100	1500—3
5—25—	100	85—17
	100	8—4
	125	
	201	
2—100—	484	
	40	
	<hr/> \$49,364	

Bought 200 hogsheads of sugar, on a credit of 12 months, weighing each 1500 lbs., gross, for which I pay  $2\frac{1}{2}$  cents per lb., net, the deduction for tare being 9 per cent.; how must I sell it, so as to realize 80 per cent profit, being allowed 5 per cent. discount?

Here, instead of saying in the discount, that 105 is reduced to 100, and using another 100 on the left, from which to advance to 80 per cent., we say that 105 discount is advanced to 180 profit.

	1	200
	100	1500—5
	100	13—
	25	
2—21—	105	
	180	
	<hr/> \$11700,00	

What would the sugar be sold for, in Havre, after deducting 20 per cent. for duty, from the prime net value?

	200	Hhd.	1	200	Hhds.
100	1500—5	G.	100	1500	Lbs., gross.
	100—13	Net.	191		Net.
	25		25		Cts. per lb.
2—21—	105	Dis.	105		
	80	M. D.	100	80	Minus duty.
	100		180		Profit.
	180				
<hr/> Ans. \$9360,00					

Above, we find first the number of lbs., then de-

duct the tare; find the cost of the whole at the price per lb.; deduct the discount, and instead of reducing the 105 to 100, merely reduce it to 80, which is allowing for the 20 per cent. duty. It is not necessary to use the two one hundreds which come between the 105 and 80, in which 105 is reduced to 100, and 100 again to 80. The duty being now deducted, and giving the net cost, we advance 80 per cent., and find the selling price, which is \$9,360. This answer might be obtained in francs, by placing 93 cents opposite the  $\frac{4}{5}$  cents, and 5 francs on the right under the  $2\frac{1}{2}$ . Although the discount would then be made on the francs, the question would be the same; as the value would not be changed.

**DIRECTIONS FOR AD VALOREM DUTIES, TARE AND TRET, ETC.**

*To find the ad valorem duty, make the whole sum the demand; 100, the same name; and the rate duty, the term of answer. The answer will be the duty in dollars, or dollars and cents, according to the denomination of the sum.*

*Or, Multiply the whole sum by the rate duty, and strike off two or more figures for cents, as the case may be.*

**FOR TARE AND TRET:** *Place the sum on the right; the standard on the left; and the standard, reduced by the tare, on the right. The answer will be the net weight.*

**FOR DUTY:** *Place 100 on the left, and the rate duty on the right: the answer will be the duty in the denomination of the sum.*

**TO MAKE DISCOUNTS, PROFITS, LOSSES, ETC.:** *Place 100, increased by the rate discount, on the left; and*

100, on the right: also, 100 on the left, and 100, increased by the gain per cent., or reduced by the loss per cent., on the right. If there be discount, and profit or loss, both in the same question, suspend the two one hundreds.

COMMERCIAL EXCHANGE.

Operations in this department of numbers are identical with those of Conjoined Proportion; hence, the statements are simple and easy, the demand being placed on the right, the same name on the left, and its equivalent on the right, as the term of answer. Again, this term of answer becomes a new demand, while a term of the same name is placed opposite, and its equivalent in value again on the right. This order is, however, frequently interrupted, by the introduction of *discounts, gains and losses, reductions, etc.*; yet such may be easily interwoven and combined in the statement by the reflecting student.

Bought 2,200 lbs., gross weight, of wool, and was allowed a deduction of 5 lbs. per 105 for tret; I paid for net weight 3 s. 6 d. per lb., New York currency, and having a credit of 12 months, was allowed a discount of 10 per cent. for ready money; for how much did I afterward sell the whole to gain 20 per cent. on my investment?

<del>2</del> — <del>1</del> <del>5</del> — <del>1</del> <del>0</del> <del>5</del> — <del>2</del> <del>2</del> <del>0</del> <del>0</del>	G. 105	2200	Gross.
1100—5	Net. 1	100	Net.
27	- 27		Shillings.
4—\$1	Sh. 8		
<del>1</del> <del>1</del> <del>0</del> — <del>1</del> <del>2</del> <del>0</del>	Dis. 110	1	Dollar.
Ans. \$1000	Par. 100	100	Par.
		120	Profit.
		\$1,000	

Above, what will 2,200 gross be reduced to, if 105.

gross = 100 net; and 1 lb. net =  $\frac{7}{8}$  shillings; and 8 shillings = \$1; and \$110, in discount, is reduced to \$100 par; and \$100 par, opposite, is advanced to \$120, 20 per cent. profit? In the solution, the two 100s, par of discount, and par of profit, were left out; and 110, discount, simply advanced to 120, profit.

Bought 1,500 lbs. of butter, at 7 d. 2 far. per lb., New Jersey currency, and was allowed 30 lbs. per 100 for firkin, and 4 lbs. per 100 for impurities or tret. I had a credit of 1 year, at 5 per cent. interest, but paying the cash, was allowed 5 per cent. discount. I immediately sold the same so as to realize 60 per cent. on my money invested: what did I get for the butter, in Federal money

100	1500
100	70
200	8
12	15
5	208
3	160
15	105
<hr/>	
	\$128

G.	100	1500	Gross.
Sut.	100	70	Suttle.
Net.	196		Net.
	2	15	Pence.
D.	12		
Sh.	20	1	Shilling.
£'s.	3	1	£.
Dis.	105	8	Dollars.
Par.	100	100	Par.
		160	Profit.
<hr/>		\$128	Ans.

A merchant has 20,000 lbs. of cotton, which he can sell at 4 d. per lb., New England currency. Failing to find a purchaser, he gives to A, in barter,  $4\frac{1}{2}$  lbs. cotton for 15 lbs. of butter: he then barter with B, giving him 40 lbs. of butter for 3 yards of gambroon: again, he barter all of his gambroon with C, giving him  $2\frac{1}{2}$  yards gambroon for 2 yards broadcloth: now, he barter his broadcloth with D, giving him  $1\frac{1}{2}$  yards for 12 yards linen; and to E he gives 30 yards linen for 8 cwt. sugar: he now barter his sugar with F, giving 3 cwt. of sugar for 50 gallons of

melasses: after this, he gives G  $4\frac{1}{2}$  galls. melasses for  $1\frac{1}{2}$  galls. of rum: he gives to H 400 gallons of rum for 3 horses: and, finally, to J he gives 2 horses for 120 sheep: he sells his sheep at \$1,80 cts. per head; how much is he gainer or loser by trading, instead of taking the original offer for his cotton?

Lbs.	920,000	Lbs.		\$20,000
	2			2
B.	4015	Butter.		4015
Ga.	53	Gambroon.		53
	2			2
C.	62	Yds. cloth.		62
	5			5
L.	3012	Linen.		3012
Su.	38	Cwt. sugar.		38
M.	950	Galls. mel.		950
	2			2
	23	Galls. rum.		23
R.	400			400
H.	23	Horses.		23
S.	1120	Sheep.		1120
	180	Cents.		180—2
\$48000,00		Ans. \$48000,00		
120000				
3—124				
201				
31				
10,00				
Ans. \$1111,11½		1111,11½		

Ans. \$46888,88 $\frac{1}{2}$

In the last calculation, £3 equal \$10; hence, the answer above; which, subtracted from the amount obtained in exchange, leaves for the gain of the merchant in trading \$46,888,88 $\frac{1}{2}$  cents. The answer of the

first question is cents, because the price of one sheep, last on the line, is cents.

From the foregoing we deduce the following

#### DIRECTIONS FOR COMMERCIAL EXCHANGE.

*Make the gross quantity the demand; the specific quantity the same name; and the specific quantity of the article which it equals, the term of answer: repeat the process, and continue the concatenation of statement, until the last term of answer is placed on the right; and the answer will be in the denomination of such last term.*

*When tare, tret, or other per cent., is to be deducted, place the standard, whatever it be, on the left; place the same standard, reduced by the deduction, etc., on the right, foruttle, net weight, etc.*

*When a discount is to be deducted, place the amount, 100 and rate, added, on the left; and 100, on the right, for present worth:*

*When a given per cent. is to be gained or lost, place 100 on the left, and 100, increased by the gain, or reduced by the loss per cent., on the right; and the answer will be the advanced or reduced price.*

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#### DECIMAL FRACTIONS.

Before commencing the article on Mensuration, it may be well to give some few general remarks on decimal fractions, that the pupil may be prepared to use them intelligibly, as they constantly occur in this department of numbers.

Decimal fractions being of little service to the ordinary arithmetician, except in Multiplication, Division, Addition, and Subtraction, we shall give only such an outline of their nature and relations as will meet the wants of the practical calculator. Consequently, the reader will not look for an elaborate explanation of abbreviations in decimal calculations; of circulating decimals; or even of the four divisions mentioned.

Units are divided into *regular* and *irregular*

fractions. When divided into 3ds, 4ths, 9ths, 16ths, etc., they are called irregular or *common* fractions; having such denominator as indicated by the number of equal parts into which the unit is divided.

In decimal fractions the unit is divided into ten equal parts; while, again, one of the latter is divided into 10 parts, making tenths, hundredths, thousandths, etc. Hence, the name decimal, from the Latin, *decem*, *ten*.

If units increase in a tenfold ratio, from right to left, certainly, from left to right, they decrease again in the same ratio. Now, continuing this decrease, from the units place to the right, it is palpable, that numbers decrease in each successive order, in the ratio of  $\frac{1}{10}$ ,  $\frac{1}{100}$ ,  $\frac{1}{1000}$ , etc., without limit. Hence, the first figure to the right of units, is ten times smaller than units; the second one hundred times smaller; the third, one thousand times smaller, and so on.

The point (.) is placed between whole numbers and decimals to distinguish them; and is called the *separatrix* or *decimal point*. The denominators of the decimals .3, .4, and .07, would be  $\frac{1}{10}$ ,  $\frac{1}{10}$ ,  $\frac{1}{100}$ . Hence, if the decimal numerator belong to an order of decimals below tenths, a sufficient number of ciphers must be prefixed to such numerator, to supply the place of the vacant orders. In the case of  $\frac{7}{1000}$ , above, it is necessary, in showing that the 7 occupies the hundred's place, to place a cipher before it, to fill the tenth's place. If the 7 were  $\frac{7}{10000}$ , three ciphers would be placed at the left for this purpose, and would be written, thus, .0007, with the cipher prefixed as far as the order of tens. From this we see, that

*The denominator of any decimal fraction is a unit, with as many ciphers annexed, as there may be figures in the numerator.*

This is reasonable when we reflect that the denomi-



Ciphers annexed to decimals do not change their value; as the significant decimals occupy still the same order or value in relation to the unit's place.

In decimal fractions, the denominator is never written or expressed; and is only understood. A great advantage in the use of decimals is, that instead of multiplying or dividing by the denominator, as many figures may be cut off, as there are tens in the denominator.

We give a few examples in the

ADDITION OF DECIMALS.

Add 318.972; 4.38; 62.7895; and 3412.013; thus,

All of the units in the whole numbers are written in a column; 8, 4, 2, and 2. At the right of this, the decimals are written, tens under tens, hundreds under hundreds, etc., each order under its separate column, and under a similar order.	318.972 4.38 62.7895 3412.013 <hr style="border: none; border-top: 1px solid black;"/> 3798.1545
---	--

We add, as in other cases, beginning at the right, and carrying all that may be over nine to the next figure at the left, both in the decimals and the whole numbers. After this, the separatrix is placed in the sum, in its own column, under the similar *separatrices* in the sums above. Therefore,

TO ADD DECIMALS:

*Place down the several whole numbers and decimals, units under units; tenths under tenths; hundredths under hundredths, etc.: add as in whole numbers, and place the separatrix of the sum under the separatrices above.*

## SUBTRACTION OF DECIMALS.

Subtraction in decimals is performed as in case of whole numbers. Let the smaller number be written under the larger; units under units; tenths under tenths; hundredths under hundredths, etc.

From 972.3856 subtract 298.534; thus,

$\begin{array}{r} 972.3856 \\ 298.534 \\ \hline 673.8516 \end{array}$	Nothing being under the 6 at the right hand, we say, 0 from 6 leaves six: 4 from 5, one; 3 from 8, five; 5 from 13, eight; 9 from 12, 3, and so on; borrowing as in the subtraction of integers. Hence,
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## TO SUBTRACT DECIMALS:

*Place the smaller of the two numbers under the larger; units under units; tenths under tenths; hundredths under hundredths, etc.: subtract as in whole numbers, and locate the separatrix, as in addition of decimals.*

If there be a larger number of decimals in the lower than in the upper number, ciphers may be annexed, *ad infinitum*, to the decimal in the upper number. We have seen before, that ciphers thus added, do not change the value of the number of decimals to which they are appended.

## MULTIPLICATION OF DECIMALS.

Multiply .46 by .5

We proceed as in ordinary multiplication; thus,

$\begin{array}{r} .46 \\ .5 \\ \hline .230 \end{array}$	If a whole number be multiplied by a decimal, the answer will be a whole number and a decimal combined; if a decimal be multiplied by a decimal, the product, according to the laws of multiplication, must be decimals only; for decimal factors cannot produce integers; nor can integers produce decimals; that is, the product must be
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of the denomination of the multiplicand and the multiplier. Hence, as both of these, in the question above, are decimals, the product must be decimals; and as there are no integers, there can be no integers in the result. Therefore,

*As many figures must be cut off for decimals, as there are decimal factors, or places in both the multiplicand and multiplier.*

As ciphers, appended to decimals, have no value, the cipher in the result above may be dropped, and the decimal called .23 hundredths, instead 230 thousandths, which is equivalent, as before.

Multiply 275.437 yards of cloth by 3.07 dollars per yard; thus,

Here, both the multiplicand and multiplier have both integers and decimals: hence, there are both integers and decimals in the answer. Cutting off five places for decimals, the answer is \$845.59159; or 59 cents and 159 thousandths of a cent. Hence,*	<div style="text-align: right;">275.437</div> <div style="text-align: right;">3.07</div> <hr style="width: 100%;"/> <div style="text-align: right;">1928059</div> <div style="text-align: right;">826311</div> <hr style="width: 100%;"/> <div style="text-align: right;">845.59159</div>
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\*SHORT METHOD OF MULTIPLYING DECIMALS.

In cases where the multiplier is 10, 100, 1000, 10000, etc., the decimal point may be removed as many figures to the right, as there are ciphers in the multiplicand. Thus: Multiply 198.7486 by 100. Thus, 19874.86, *Ans.*

Decimals below the 4th and 5th orders, are so small as to be of very little value: hence, when the decimal places are very numerous in the multiplicand and multiplier, or either, and it is not desired to extend the calculation beyond 4, 5, or 6 decimal places, it becomes necessary to resort to a method of finding the product without multiplying to whole number of decimals. This is done as follows: We assume the following, in ordinary multiplication:

The left hand figure of the multiplier may be multiplied by

## TO MULTIPLY DECIMAL FRACTIONS.

*Proceed as in the multiplication of whole numbers; cutting off as many figures in the product for decimals, as there may be decimal factors or places, both in the multiplicand and multiplier.*

*When there is a larger number of decimal places*

first, if tens are still placed under tens; hundreds under hundreds, etc.; thus,

$\begin{array}{r} 1284 \\ 2475 \\ \hline 2568 \\ 5136 \\ 8988 \\ 6420 \\ \hline 3177900 \end{array}$	$\begin{array}{r} 1284 \\ 2475 \\ \hline 2568 \\ 5136 \\ 8988 \\ 6420 \\ \hline 3177900 \end{array}$
--	--

Hence, as above, commencing with the left hand figure in the multiplier, and causing the scale to descend to the right, produces the same result, as multiplying first by the unit's place and descending to the left.

Now, it is evident, that we may multiply in cases of decimals in the same way, and by carrying the multiplication to a certain number of orders to the right, to get the product of the orders so used, throw away all useless and minute multipliers.

Let us multiply 2.8724 by .37854; thus,

2.8724	
.37854	
8617 2	
2010 6 8	
229 7 92	
14 3 620	
1 1 4896	
1.0873 1	8296 Ans.

All of the figures to the right of the vertical line, are useless, as there are five places of decimals on the left of it, being as many decimals as desired in the product.

In placing down the entire products, the figures on the right of the line serve to show what numbers are carried to the first place of decimals retained on the left.

## MULTIPLICATION OF DECIMAL FRACTIONS. 207

*in the multiplicand and multiplier, than in the product, prefix ciphers to the product until the deficiency is supplied.*

We may now show how to get the figures on the left of the line, without having to make those on the right.

We multiply the first right-hand figure of the multiplicand, by the left-hand figure of the multiplier, placing the product under the figure thus multiplied. We next multiply by the second decimal multiplier, 7. Multiplying this into 4 would necessarily cause the product to be placed one move to the right of the former product, 2; and, as this is unnecessary, we multiply the 7 into the second figure from the multiplicand, 2, carrying to the product the nearest number of decimals which this 7 and the suspended 4 would make. Seven times 4 making 28, nearly 3 decimals, we say, 7 multiplied by 2 equals 14, and 3 added, makes 17. Hence, the 7 is placed under the 2, at the right, and 1 carried to the product of 7 into 7, which makes 49, making it 50, and so on. We multiply again by the next decimal multiplier, 8, suspending both the 4 and 2, at the right of the multiplicand, and carry its product into the 7 above, the nearest number of decimals that the 2 and 4 make. Thus, 8 times 7 are 56: now, 8 times the former 2 are 16, and 8 times 4 are 32; carrying 3 from 32, to 16, makes 19; very nearly 2 decimals: hence, 2 added to 56 make 58; the 8 being placed under the right-hand column, and the 5 carried as before. In like manner we proceed next with the 5 and 4. After multiplying 8 by 5, we carry 3; because 5 into 7, the suspended order, makes 35, which we call only 3 decimals, although it is three and a half, allowing this half over 3, for the deficit in previous numbers, where the number of decimals carried, was rather too large. Thus, the numbers become pretty well balanced.	$  \begin{array}{r}  2.872'4 \\  \times 7854 \\  \hline  86172 \\  20107 \\  2298 \\  143 \\  11 \\  \hline  1.08731 \text{ Ans.}  \end{array}  $
--	---

Above, we must always cast off one figure less in the multiplicand than the number of decimals which we wish to retain: for the first decimal figure of the multiplier, when multiplied by, would otherwise give one factor too many; and consequently, one place of decimals too many in the product. The first multiplier will, however, be multiplied into the rejected figure, and the nearest number of decimals in the pro-

## DIVISION OF DECIMAL FRACTIONS.

How many gallons of molasses, at .4 of a dollar per gallon, can be bought for .96 of a dollar?

Here, .4 equals  $\frac{4}{10}$ ; and .96 equals  $\frac{96}{100}$ . We now divide the latter common fraction by the former; thus,

$$\begin{array}{r} 10\cancel{0}\overline{)24} \\ \underline{40} \\ 24 \end{array}$$

The answer is in units of gallons. Now, instead of dividing 24 by this 10 on the left, it is quite as easy to cut off one figure at the right of the 24; thus, 2.4, and the 4 is understood as  $\frac{4}{10}$ , or a decimal. Hence, it may be divided decimally; thus,

$$\begin{array}{r} .4\overline{) .96} \\ \underline{.24} \end{array}$$

In this division, the decimal on the left, equals or neutralizes one decimal on the right: hence, the remaining decimal becomes a unit.

In multiplying decimals, the *product* must have as many decimals as there are decimals in both the multiplicand and multiplier. The *dividend is always equal to the product of the divisor and quotient*; hence there must be as many decimals in the divisor and quotient, taken together, as there are in the dividend. Therefore,

*To locate the decimal point, ascertain the difference between the number of decimals in the divisor and the dividend; the remainder will be the number of decimals to be cut off, in the quotient. If there are not as many decimal places in the quotient as the difference, prefix ciphers to the quotient, until the num-*

duct, will be carried to the product of the first figure of the multiplicand. The rejected figure is in the column of the last decimal of the answer. Hence, count to the left from this, and locate the decimal point accordingly.

This subject cannot be treated elaborately here; as it belongs to *Elementary Arithmetic*.

ber in the quotient equals the number in the difference.

Divide .00954 by 3.08.

$$\begin{array}{r} 3.08 \overline{) .00954(.003} \\ \underline{924} \\ 30 \end{array}$$

The answer is 3 thousandths. By annexing ciphers to the 30, the division may be continued, and the quotients placed at the right of

the .003.

However far the division may be carried in this example, it will not terminate. It is called a *circulating decimal*. The division has been continued on to fourteen places, giving the following,

.00309740292207792

When the number of decimals in the dividend and divisor, is the same, the quotient is a whole number. When there are not as many decimals in the dividend as in the divisor, annex ciphers to the former until they are equal.

It is generally very useless to carry decimal calculations further than four or five figures

#### TO DIVIDE DECIMAL FRACTIONS.\*

*Proceed as in the division of whole numbers; and*

#### \*CONTRACTION IN THE DIVISION OF FRACTIONS.

Division of decimals may be very much abridged when there is a very large number of decimal places in the divisor, as in the following example.

Divide 4.3125 by 3.2364, retaining four decimal places in the answer.

*Common Method.*

$$\begin{array}{r} 3.2364 \overline{) 4.3125} \quad (1.3324 \\ \underline{3 \ 2364} \\ 1 \ 0761 \ 0 \\ \underline{9709} \ 2 \\ 1051 \ 80 \\ \underline{970} \ 92 \\ 80 \ 880 \\ \underline{64} \ 728 \\ 16 \ 1520 \\ \underline{12} \ 9456 \\ 3 \ 2064 \end{array}$$

*Contraction.*

$$\begin{array}{r} 3.2364 \overline{) 4.3125} \quad (1.3324 \\ \underline{3 \ 2364} \\ 1 \ 0761 \\ \underline{9709} \\ 1052 \\ \underline{971} \\ 81 \\ \underline{65} \\ 16 \\ \underline{13} \\ 3 \end{array}$$

*out off, in the quotient, a number of figures equal to the excess of the decimals in the dividend, compared with those in the divisor. If the number of decimals in the quotient be too small, prefix ciphers until the number equals the excess, above named.*

Reduce the decimal .225 to an equivalent common fraction of the lowest term.

Subscribing the denominator, .225 becomes  $\frac{225}{1000}$ . This reduced to its lowest term is  $\frac{9}{40}$ . Hence,

#### TO REDUCE DECIMALS TO COMMON FRACTIONS.

*Cancel the decimal point, and place the denominator below the given decimal; reduce the fraction to its lowest term, and the answer will be an equivalent common fraction, in its lowest term.*

The first figure of the quotient is 1. Now, instead of annexing a cipher to each remainder, and thus multiplying it successively by 10, we reject at each separate division the right-hand figure of the divisor, which is equivalent to dividing it successively by 10. In multiplying the last unrejected figure in the divisor, 6, by the second quotient figure, 3, making 18, we carry 1, which is the nearest decimal that the product of the 3 and the rejected 4, will make. The decimal accession, from the rejected figures of the divisor is considered in each subsequent multiplication and division, until the 4 required decimal orders are found for the quotient. In multiplying the 3 in the divisor, by the third 3 of the quotient, the product is increased to 11 by the accession from the last two rejected figures, 3 times 6 making 18, and 3 times 4 making 12, the sum of which is 20, or 2 decimals.

If the divisor has more figures than the number required in the quotient, including integers and decimals, take as many on the left of the divisor as required in the quotient, and divide by them, as in other cases.

If the number in the divisor be smaller than that required in the quotient, divide as ordinarily, until the deficiency is filled; after which, contract as before.

When the divisor is 10, 100, 1000, etc., remove the separatrix to the left, as many places as there are such ciphers; and the division will be performed without further reckoning.

## COMMON FRACTIONS REDUCED TO DECIMALS. 211

Reduce  $\frac{3}{4}$  to an equivalent decimal fraction.

We multiply the numerator and the denominator, each, by 100; because the product of two tens into the numerator, is the smallest number that can be divided by the denominator, 4. This makes  $\frac{300}{400}$ , which divided by the original denominator, 4, gives  $\frac{75}{100}$ . This  $\frac{75}{100} = .75$ ; for always dividing the product of the numerator and denominator into tens, hundreds, etc., by the same denominator thus multiplied, it is evident that the denominator must always be composed of tens. Since, therefore, these tens, hundreds, etc., in the denominator of a decimal, are useless, we avoid the process of getting them, and simply annex ciphers to the numerator of the common fraction, and divide by the denominator, until an exact result is obtained, or as many decimal places as requisite; Thus,

The division here terminates in two places  $\begin{array}{r} 4 \overline{)300} \\ \underline{280} \\ 20 \\ \underline{20} \\ 0 \end{array}$   
of annexed ciphers. Again: .75

Reduce  $\frac{1}{2}$  to a decimal,

Here the divisor terminates in one place of  $\begin{array}{r} 2 \overline{)10} \\ \underline{10} \\ 0 \end{array}$   
annexed ciphers. .5

Reduce  $\frac{2}{3}$ \* to a decimal; thus

$$\begin{array}{r} 3 \overline{)20000000000000000000} \\ \underline{6666666666666666666} + \end{array}$$

### TO REDUCE A COMMON TO A DECIMAL FRACTION.

*Append ciphers to the numerator, and divide by the denominator, until the denominator terminates, or*

\* This is called a repeating decimal; showing that although the process might be repeated *ad infinitum*, yet the true result would never be obtained. Hence, although we get nearer to the true answer at every step, yet, we would never get it entirely, although the division were continued forever. In such cases the division need not be carried further than from four to six places, as in the seventh place one of the sixes would be only  $\frac{6}{1000000}$ , a very minute and scarcely conceivable common fraction. The plus mark is appended to show that it is still imperfect.

until a sufficient number of decimals is obtained. Cut off, for decimals, in the quotient, a number of places equal to the number of ciphers annexed.

When the figures in the quotient are not equal to the number of ciphers annexed, prefix ciphers to the quotient, until the deficiency is supplied.

The method of pointing off above, will appear reasonable, when we reflect, that every cipher annexed to the numerator, multiplies it by 10; hence, after it is divided by the denominator, the quotient will be ten times too large, and should, consequently be divided again by 10. This is done most easily, by cutting off one figure toward the left. The same reasoning is true as regards annexing two, three, or more ciphers, and increasing in the multiplication, by 100, 1000, etc., necessitating a division of the result by the same numbers. Hence, the propriety of striking off a number of figures in the result, for decimals, equal to the number of ciphers, appended.

To reduce compound numbers to decimals, Reduce the denominate number to a fraction of the denomination required, and this fraction, to a decimal.

Reduce 5 shillings 3 pence to the decimal of a pound.

5 s. = 60 d; and 60 ÷ 3 = 63 d: now, £1 = 240 d: hence  $\frac{63}{240}$  of a pound must now be reduced to a decimal, thus,

$$\begin{array}{r} 24 \overline{) 63000(.2625} \\ \underline{48} \phantom{000} \\ 150 \phantom{00} \\ \underline{144} \phantom{00} \\ 60 \phantom{00} \\ \underline{48} \phantom{00} \\ 120 \phantom{00} \\ \underline{120} \phantom{00} \\ 0 \end{array}$$

If we divide by the 240, there would be a cipher in the unit's place; but dropping the cipher in the divisor, we have no unit in the quotient, and place the decimal point at the left of the answer, which is .2625 decimals of a pound.

Reduce 15 minutes, 30 seconds to the decimal of an hour.

15 × 60 = 900 ÷ 30 = 930 seconds: now, 1 hour contains 3600 seconds: hence, reduce  $\frac{930}{3600}$  to a decimal; thus,

$$36 \overline{) 9300000(.258333}$$

The result is .258333. This is a repeating decimal; hence, the calculation is discontinued at six places.

All other reductions in denominate numbers may be made as these.

## MENSURATION, OR PRACTICAL GEOMETRY.\*

MENSURATION is that department of the science of numbers, which treats of the measurement of lines, superficies, solids, &c., and is derived from *mensura*, *measure*.

The general principles and laws regulating this department of the science, are derived from Geometry.

Geometry is the science of magnitude, in all its various forms and relations; and is divided into practical and theoretical. The latter treats of those portions which are so complex as to require symbols and the higher mathematical formulæ for their illustration: the former treats of such portions only, as depend on the simple relations of numbers, as manifested through proportion.

Geometry is from *γη*, *the earth*, and *μετρον*, *measure*, and primarily signified the *measurement of the earth*.

Many of the laws of Geometry are demonstrated by formulæ that the ordinary reader

\* It may be remarked, while treating of superficial measurement, that *Abacus* is a Latin word which means *flat*. In the primitive ages, all calculations were made by the Oriental nations on boards covered with *dust*, on which lines and signs could be easily traced. From the word *abak*, signifying *dust*, the Greeks deduced their word *αβαξ*. This Abacus used by the Romans, and Abax by the Greeks, was a large board with transverse lines drawn on it, on which calculations were made by sundry movements of *pebbles*, or *calculi*. Hence, the derivation of our English word *calculate*, from *calculo*, which is from *calculus*, a pebble. The latter word is from the Syriac *kalkai*, *gravel*.

would not comprehend; they can, however, be made quite as intelligible by the manifest relations and deductions of common sense, without the exercise of which, all formulæ become mere mechanical arrangements; being neither appreciated nor understood. Too many writers endeavor to teach Mensuration by the introduction of Geometrical signs and reasonings; thereby endeavoring to teach a primary, by the *rules* of a secondary science. Hence, the reason of so many failures in this study; and hence, the mechanical patch-work by which many practical men make such calculations.

Practical Geometry is divided into *superficial* and *solid*. *Superficial*, which is from *superficies*; *the surface, the outside, &c.*, relates to the *measurement of surface, which has extent merely, without bulk; and has two sides given to find the contents*: *Solid Geometry* relates to the measurement of bodies or magnitudes, which have *length, breadth, and thickness*. This species of measurement is generally called *cubic*. A cubic foot of timber, is a foot *long*, a foot *wide*, and a foot *thick*, or 12 inches in every way: hence, when these 3 twelves are multiplied continuously, they make 1728, the number of cubic inches in a cubic foot.\*

Cubic is from the Latin *cubicus*, from *cubus, a die*. - Hence, cubic is a congregation of particles, forming a solid mass of six equal sides.

According to the laws of Multiplication, concrete objects cannot be multiplied together;

\* This solid foot, or 1728 cubic inches, weighs 1000 ounces rain water.

nor can concrete objects of different denominations be multiplied. Feet multiplied by feet, through ratio, will give feet; but feet multiplied by inches, will give neither feet nor inches. Ten feet long and 12 inches wide will give neither 120 feet nor 120 inches. Hence, *when the denominations are dissimilar, such reductions must be instituted as will make the terms alike.* Above, if we divide the width 12, by the number of inches in a foot, we find that the width is 1 foot; now, the length and width being in feet, we conclude that there are 10 superficial feet.

Ten feet in length and 10 in width give 100 feet superface: this multiplied by 10 feet in height, will give 1000 feet solidity.

*These similar dimensions, length, height, and width, multiplied together, give the cubic or solid contents of the figure, in the denomination of the dimensions: as a crib, a box, a wall, a boat, a cistern, &c.*

Let the learner keep these truths before his mind, and but few difficulties will present themselves in ordinary measurements. Whenever mathematical rules are introduced, they must be received by the student on authority, as it would be impossible, in a treatise on numbers, to develop their principles.

It may be remarked here, that too much time is generally spent on algebraic and mathematical solutions, while the learner proposes to study arithmetic. The introduction of such questions and rules, is an oversight in too many authors: for the student thus wastes his time in pursuing the work of mathematics,

which is impossible in arithmetic; while it should be devoted to numbers only; for nothing else than numbers can be learned in arithmetic.

#### WOOD AND BARK.

WOOD and BARK are generally measured by the cord, which is a pile 4 feet wide, 4 feet high, and 8 feet long. The word *cord* is derived from the Welsh *cord*, signifying a twist, relating to a rope: hence, the *cord*, or *rope*, with which the ancients were accustomed to measure a pile of wood, gave 128 solid feet, which these dimensions, 4, 4 and 8 make, when multiplied. A pile of wood contains more or less than a cord, when it has more or less than 128 solid feet. Hence, wood is measured by proportion. We may multiply together the dimensions of the pile, and compare the whole number of feet with 128; or we may compare the several separate dimensions with 4, 4, and 8. The latter is preferable.

How many cords of wood in a pile 120 feet long, 20 feet wide, and 2 feet high?

$$\begin{array}{r}
 8 \overline{) 120} - 15 \\
 4 \overline{) 20} - 5 \\
 2 \overline{) 4} - 2 \\
 \hline
 37\frac{1}{2} \text{ cords.}
 \end{array}$$

Here, we place the several dimensions of the pile, on the right, and the dimensions of a cord opposite these, on the left; and say, what will all these feet on the right make, if 8, 4, and 4, on the left, make 1 cord, last on the right. It is unnecessary to place the 1 on the right, as it will not assist in the calculation.

How many cords in a pile 200 feet long,  $3\frac{1}{2}$  feet wide, and 16 feet high?

Here, we say 4 times 4 on the left, equal 16 on the right. The answer is eighty-seven and a half cords.

$$\begin{array}{r|l} 8 & 200-25 \\ \cancel{4} & 17 \\ \cancel{4} & 16 \\ 2 & \\ \hline & 87\frac{1}{2} \text{ cords.} \end{array}$$

How many cords in a pile of bark 20 feet long, 3 feet 4 inches high, and  $10\frac{1}{2}$  feet wide?

In this instance, 4 inches are  $\frac{1}{3}$  of a foot, making the height  $3\frac{1}{3}$  or  $\frac{10}{3}$  feet. The numerator of this, as well as of the  $\frac{21}{2}$ , is placed on the right, and the denominator opposite. The answer is  $5\frac{1}{3}\frac{1}{2}$  cords.

$$\begin{array}{r|l} 8 & 20-5 \\ \cancel{3} & 10-5 \\ \cancel{2} & 21-7 \\ \cancel{4} & \\ 4 & \\ \hline 32 & 175 \\ \hline & 5\frac{1}{3}\frac{1}{2} \text{ cds.} \end{array}$$

What will a load of wood 8 feet long, 2 feet 6 inches high, and 3 feet 4 inches wide, come to, at 1 dollar and 80 cents per cord.

Again, the inches are made the fractional part of a foot, and added to the given feet in each case; while the mixed number is placed on the line in the form of an improper fraction. We know that the 8, 4, and 4, on the left, make

$$\begin{array}{r|l} 8 & 8 \\ \cancel{2} & 5 \\ \cancel{3} & 10-5 \\ \cancel{4} & 180-3-5 \\ 4 & \\ \hline 4 & 375 \\ \hline & 93\frac{3}{4} \text{ cents.} \end{array}$$

one cord, or that these 128 feet are worth the price, 180 cents; then the price is placed last on the right in the place of the one cord, and the answer must be the price of the whole pile of wood at 180 cents per cord. This is nothing more than simple proportion. Twice

3 on the left, goes into 18 on the right three times. The answer is 93½ cents. This method is quite preferable to ascertaining the quantity, which may be fractional, and multiplying it by the price as a separate operation.

4—8	10—5
43	
47	
<hr/>	
64	105
<hr/>	
	1½¼
8	10—5
2—43	
47	
	240—3—5
<hr/>	
4	1575
<hr/>	
\$	3,93½
8	10—5
2—43	
420—4	
	2
<hr/>	
2	75
<hr/>	
\$	37½

How many cords in a pile of wood 10 feet long, 3 feet wide, and 7 feet high? and what will the same come to, at 240 cents per cord?

There are 1½¼ cords wood.

We will now state both in one, thus. The two was used in 4 on the left, and in 10 on the right.

What will a pile of wood 40 feet long, 3 feet high, and 20 feet wide, come to, at 2 dollars per cord?

The price is dollars in this case, and the answer is in dollars; 37 dollars and 50 cents.

From the foregoing, we conclude that,

*To ascertain the number of cords in a pile or load of wood or bark, place all of the dimensions on the right in feet, and 4, 4, and 8, or 128, on the left. If there are inches in any of the dimensions, they must be reduced to the fraction of a foot, and added to the feet, and treated as other improper fractions.*

*If the answer is desired in the price of the whole quantity of wood, place the price of one cord last on the right, in dollars or cents, and the*

*answer will be the price of the whole, in dollars or cents.*

## LUMBER MEASURE.

Under this head may be classed *superficial* board measure, and the measurement of solid timber. We have only two general dimensions in board measure; length and width. The thickness is generally considered a unit; inch measure being the standard. Anything under one inch is not noticed; but all above an inch in thickness, as two inches, three inches, &c., is called two, three, &c., thicknesses of lumber. If a piece of lumber 20 feet long, 16 inches wide, and 8 inches thick, be measured, the thickness is called eight planks.\*

The first thing to be done with such a question as the one above, is to reduce the width, which is in inches, to feet, that width and length in feet may be multiplied together for the superficial contents. This would afterwards be multiplied by the 8 thicknesses, giving 8 times as many feet as in the one piece. All of this may be done in the same operation; thus,

It will be observed here, that the length, width, and thickness are all placed on the right of the line, and 12 only, on the left, to reduce the width, 16 inches, to feet. Hence, the answer is  $213\frac{1}{2}$  feet.

3—12	20
16	16
8	8
—	—
—	640
—	—
—	213½ ft.

\* In America the words *board* and *plank*, are variously used to denote the same thing. This is incorrect. While a board is a thin piece of timber, a plank is a thick and heavy piece. The word is from the Dutch *plank*, or the Danish *planke*, a thick board. Hence the difference.



say, what will all of these feet come to, on the right, if 100 feet opposite, cost  $37\frac{1}{2}$ , or  $\frac{75}{2}$  cents? The answer is 5400, the number of dollars and cents, which pay for the whole.

Suppose in the case above the timber will lose  $\frac{3}{8}$  of an inch in sawing. We say, what will the whole quantity of lumber be reduced to, if  $\frac{1}{8}$  be reduced to  $\frac{3}{8}$ ?

	16
12	15
	6
	120
16	13
100	
2	75
<hr/>	
\$43,874	

What will 4 pieces of timber come to, at \$24 per 100 ft. which are 10, 20, 18 and 12 feet long respectively, and 16 inches wide, and 3 inches thick?

10  
20  
18  
12

60 entire length.

	60
2—12	16—4—2
2—100	3
	2
	5
<hr/>	
\$6	

In this instance, it is necessary to add the several lengths, and place their sum on the right. Had there been 10, or any other number of pieces in each pile, 10 or such number would be placed on the right, once, and only once: for the question, by getting the sum of the lengths, was changed into this, how many feet in a piece 60 feet long, 16 inches wide, and 3 inches thick? Hence, ten times the number in each case, would be ten times the 60 feet.

What will 10 piles lumber, with 40 pieces  
15

in a pile, come to, at \$11 per 100 feet, the plank being 18 inches wide, 3 $\frac{1}{2}$  inches thick, and 20, 16, 17, 19, 23, 10, 7, 12, 6, and 20 feet long? The sum of the lengths is 150 feet: hence, we place it on the right, thus,

$$\begin{array}{r|l}
 150 \\
 2-12 \overline{) 18-3} \\
 \underline{4 \ 15} \\
 100 \ 40 \\
 \underline{4 \ 5} \\
 8 \overline{) 3375} \\
 \hline
 \$ \overline{) 4217}
 \end{array}$$

Here, 40 planks in a pile, is placed down once for the whole lot, considering that the lot is now 150 feet long. The answer is in dollars, because the price was dollars.

$$\begin{array}{r|l}
 8 \\
 2-12 \overline{) 9-3} \\
 \underline{2 \ 5} \\
 100 \ 80-4 \\
 \underline{60} \\
 \$ \overline{) 7,20}
 \end{array}$$

What will 80 pieces of lumber, 8 feet long, 9 inches wide, and 2 $\frac{1}{2}$  inches thick, come to, at 60 cents per hundred?

We deem the examples given, sufficient for the measurement of lumber, as there is but very

little difficulty in the statement.

How many cubic feet in a stick of timber 30 feet long, 8 inches thick, and 10 inches wide?

$$\begin{array}{r|l}
 30 \\
 4-12 \overline{) 8-2} \\
 3-6 \overline{) 10-6} \\
 \hline
 3 \overline{) 50} \\
 \hline
 16\frac{2}{3} \text{ ft.}
 \end{array}$$

In this example it is necessary to divide both the width and the thickness by 12, to reduce them to feet, that by multiplying all the dimensions in feet, the product may be solid feet. Hence,

*To measure lumber, Place the length in feet, the width in inches, and the thickness, in inches, on the right, and 12 on the left.*

*To ascertain the number of feet in the whole pile, when of the same dimensions, place the number*

of picces, likewise on the right. If the answer is desired in dollars, or dollars and cents, place 100 on the left, and the price per hundred, on the right.

To lose a fraction for saw-cut, subtract the fraction lost from such a number of parts of the same size as would constitute a unit, place the remainder on the right, and the number making a unit, on the left.

MASONRY.

Masonry, as a department of measurements, may properly be classed with cubic timber measure. Stone work is measured by the perch, which is generally 25 solid feet, or  $16\frac{1}{2}$  feet long,  $1\frac{1}{2}$  feet wide, and 1 foot high. A solid perch in masonry, is a mass  $16\frac{1}{2}$  feet in every way. The word perch is derived from the French *perche*, which signifies *sharp, extending, &c.*, as a pole or rod for measurements. Hence the name is derived from the limits which define it.

How many perches of masonry in a wall 80 feet long, 15 feet high, and  $2\frac{1}{2}$  feet thick?

Here, we make 25 solid feet a perch, saying, what will all of the feet in the wall make, if 25 opposite make 1 perch. Hence 120 perches.

$$\begin{array}{r|l} \$-25 & 80-4 \\ & 15-3 \\ & 2\frac{1}{2} \\ \hline & 120 \text{ per.} \end{array}$$

We may easily ascertain the price for a piece of work at the same time that the quantity is obtained, by placing the price per perch last on the right.

$$\begin{array}{r|l}
 25 & 200-5 \\
 4 & 25 \\
 4 & 15 \\
 \hline
 & 120-3 \\
 \hline
 \$ & 225,00
 \end{array}$$

What will it cost to put up a wall 200 feet long, 6 feet 3 inches high, and  $3\frac{1}{2}$  ft. thick, at 120 cents per perch of 25 feet?

The price being in cents, 2 figures are cut off at the right of the answer, for cents.

How much will it cost to wall a cellar, at \$1,60 cents per perch, 20 feet square, and  $7\frac{1}{2}$  feet deep, with a wall  $1\frac{1}{2}$  feet thick?

$$\begin{array}{r|l}
 5-25 & 74-27 \\
 2 & 15-4 \\
 2 & 3 \\
 \hline
 & 1,60 \\
 \hline
 \$ & 53,28
 \end{array}$$

It is evident that the two end walls are each 3 feet shorter than those of the sides: hence, the entire length of the wall is 74 feet. We place this, with the height and thickness, on the right, and the denominators on the left. We use the factor 5 on the two sides of the line. Hence,

*To ascertain the number of perches in a piece of stone work, place the length, height, and width, in feet, on the right, and 25, or whatsoever standard is acknowledged, on the left: the answer will be solid perches. If the cost is desired, place the price per perch last on the right, and the answer will be the cost of the entire work, in dollars or cents.*

#### PLASTERERS', PAVERS', AND BUILDERS' WORK.

Plasterers and Pavers calculate their work by the square yard, or 9 square feet: Builders reckon by the square, which is 100 square

feet, in weather-boarding, ceiling, framing, shingling, &c.

How many square yards of plastering in a room 18 feet square, and 10 feet high. We place the side, 18, on the right, and 4, which will give all the sides.

Having the dimensions in feet on the right, we place 9 feet, which make a square yard, on the left: the answer is the sum of the four sides,

$$\begin{array}{r} 18 \\ 9 \overline{) 4} \\ 10 \\ \hline 80 \text{ yards.} \end{array}$$

80 yards. We now place 18 on the right of another line twice, and ascertain the number of yards overhead, by the same process, which is 36.

$$\begin{array}{r} 18 \\ 9 \overline{) 18} \\ 36 \\ \hline 80 \\ \hline 116 \end{array}$$

The two added, make the number of yards in the room, 116. We might have ascertained the cost of the whole, quite as easily, by placing the price in each statement, last on the right.

What will the plastering of a room come to, which is 15 by 20 feet, and 12 feet high, at 22½ cents per yard?

The 4 sides make 70 feet around, which we place with the height and price, on the right. The cost of the sides is \$21.00. Again, we place 15 and 20 on the right, with the price, and 9 on the left. This makes the plastering overhead come to 7 dollars and 50 cents, which, added to the sum above, makes the

$$\begin{array}{r} 70 \\ 2 \overline{) 12} \text{ } 4 \text{ } 2 \\ 245 \text{ } 15 \\ \hline \$21,00 \\ 7,50 \\ \hline \$28,50 \end{array}$$

$$\begin{array}{r} 15 \\ 2 \overline{) 20} \\ 245 \text{ } 50 \\ \hline \$7,50 \end{array}$$

cost of the room amount to 28 dollars and 50 cents.

$$\begin{array}{r|l} \$300 & -25 \\ \$0 & \\ \hline \$75 & \\ \hline \$56,25 & \end{array}$$

How much will it cost to lay a pavement 300 feet long, and 4 feet 6 inches wide, at  $37\frac{1}{2}$  cents per square yard?

This question is identical with those just wrought in plastering. Hence,

*To ascertain the number of yards of plastering, or paving, place the whole length of the walls, or pave, with the width or height in feet, on the right, and 9 on the left: if the answer is wished in money, place the price per square yard, last on the right: the answer will be the price of the whole.*

How many squares of weather-boarding on a building 50 by 40 feet, 21 feet high? What will the same come to, at \$1,50 per square?

$$\begin{array}{r|l} 180 & \\ \$00 & 21 \\ 1,50 & \\ \hline \$46,70 & \end{array}$$

The whole length of the building, or sum of the sides, is placed on the right, with the price, and 100, the number of feet in a square, on the left.

What does it cost to shingle a roof 80 feet long, and 20 feet from the eaves to the cone, at  $87\frac{1}{2}$  cents per square?

$$\begin{array}{r|l} 80 & \\ \$00 & 40-2 \\ 2175 & \\ \hline \$28,00 & \end{array}$$

The roof is 80 by 40 feet, which dimensions are placed on the right, with the price. Thus, by proportion, what will all of these feet come to, on the right, if 100

feet, opposite, cost  $87\frac{1}{2}$  cents? The answer is \$28. Hence,

*To ascertain the cost of weather-boarding, shingling, framing, flooring, &c., place the length and width in feet, on the right; 100 on the left; and the price per square, last on the right.*

## CRIBS, BOXES, AND BODIES.

The standard of measurements of this kind, is generally inches; the number of inches making a bushel, a gallon, &c.

A compact bushel, as wheat, shelled corn, salt, &c., contains 2150 $\frac{1}{2}$  cubic inches, which may be expressed and used decimally, in the form of 2150.2.

A dry bushel, as potatoes, apples, coal, &c., contains 2688 cubic inches.

A wine gallon contains 231: a beer gallon, 282 cubic inches.

A solid foot contains 1728 cubic inches.

If the length, width, and height of a body, crib, or box, are placed on the right of the line, their product will be the number of cubic inches in such body, crib, &c. The question then is, by proportion, what will all these inches on the right give, if 2150 opposite, give 1 bushel; or, if 2688 give one dry bushel; or, 231 give 1 wine gallon; or, if 282 give 1 beer gallon, &c.? Hence, after these dimensions are placed on the right, it is only necessary to place the number making a unit of the desired measure, on the left. If, in measuring coal, the body is wider at the top than at the bottom, take the *mean width*, by measuring half way between the top and bottom.

The 2150 inches used, make an *even* bushel; hence, when the corn is in the ear, place 10 on the left: when in the husk, place 20 on the left, and the answer will be in barrels. This allows 5 bushels of shelled corn, 10 of unshelled, and 20 in the husk, for a barrel.

When it is necessary to ascertain the whole price, the price per bushel, &c., may be placed last on the right.

How many bushels corn in a crib 160 inches long, 86 inches wide, and 90 inches high?

The factor 5 is here used in the 10, and 215.

As—2150	160
	86—2
	90—2
	576 b.

How many bushels in a crib 120 inches long, 45 wide, and 80 high?

We frequently find it impracticable to cancel in these calculations. This, however, is of but little moment.

As—2150	120
	45—9
	80
	8640
	200 $\frac{1}{2}$

$$\begin{array}{r|l}
 200 & \\
 2150-2150 & 100 \\
 & 50 \\
 \hline
 & 2150 \\
 \hline
 & \$100.00
 \end{array}$$

$$\begin{array}{r|l}
 2688-2688 & 120 \\
 22 & 45-3 \\
 4 & 60-15 \\
 \hline
 & 27 \\
 \hline
 & \$4.50
 \end{array}$$

$$\begin{array}{r|l}
 70 & \\
 22-221 & 60-2 \\
 2 & 55-5 \\
 \hline
 & 1000 \text{ galls.}
 \end{array}$$

$$\begin{array}{r|l}
 120-4 & \\
 141-282 & 40-2 \\
 47 & 20 \\
 & 50 \\
 \hline
 & 800000 \\
 \hline
 & 170,21\frac{1}{4}
 \end{array}$$

What will a crib of corn come to, at  $21\frac{1}{2}$  cents per bushel, 200 inches long, 100 wide, and 50 deep?

This combination of statement is quite simple and easy.

What will a load of coal cost, at three and a half cents per bushel, which measures 120 in. long, 48 inches wide, and 60 inches high?

The price, 4 dollars and 50 cents, pays for the whole load.

How many gallons water in a tan vat, 70 inches long, 60 inches wide, and 55 inches deep?

The answer is 1000 gallons, in measure.

How much will a vat of beer come to, at 50 cents per gallon, the vat being 120 inches long, 40 inches deep, and 20 inches wide?

The answer is 170 dollars, 21 cents, and a fraction.

To find the contents of a crib, body, or box, in compact bushels, place all the dimensions on the right, in inches; and on the left 2150. If the answer is wished in barrels, place 5 on the left for shelled corn, 10 for corn in the ear, and 20 for corn in the husk.

When the answer is desired in dry bushels, place 2688, instead of 2150, on the left. In either case, place the price, on the right, last, and the answer will be the price of the whole.

To ascertain the number of gallons, place the dimensions, as above, on the right, and for wine gallons 231, or for beer gallons 282, on the left. The price per gallon may be placed last on the right; the answer will be the price of the whole.

When the contents of any crib, box, or body, and two of the sides are given to ascertain the other side, place the contents and the standard of unity, or the number of inches which make a unit of the contents, on the right, and the two given dimensions on the left: the answer will be the required side.

A body is 120 inches long, and 36 inches wide; how high must it be to hold 180 bushels of coal?

It is seen here, that the  
body must be 112 in. high; or  
9 ft. 4 in. Again,

2—120	2688—224
2—36	180—15—3
	112 inches.

A crib is 215 inches long, and 100 inches wide; how high must it be to hold 1500 bushels of corn?

Thus, the crib must be 150 in.,  
equal to 12½ feet high.

215	2150
100	1500
	150 in.

## TONNAGE OF VESSELS.

In ascertaining tonnage, it is necessary to ascertain as nearly as practicable, the number of cubic feet of water displaced by the vessel. This is done by multiplying together the length, width, and depth of the vessel, which gives the number of cubic feet contained in the hull. Now, it is a law of hydrostatics, that "*if a body floats on a fluid, it displaces as much of the fluid, as is equal to its own weight.*" Nor does it make any difference what the shape of such body be; a quantity of water equal to its own weight must be displaced. Hence, in ascertaining the weight that any hollow square will sustain in water, it is necessary, first, to ascertain the weight of water such square would contain; making all due allowance for weight of vessel, room for safety, etc., etc. A cubic foot of water weighs 1000 ounces *avoirdupois*,\* or 62½ lbs.; hence, 95, the number of cubic feet allowed

\* The cubic foot of distilled water weighs about 1000 oz. *avoirdupois*, or very nearly 62½ lbs., at 40° temperature; at 60°, which is generally used, it weighs only 62,353 lbs., less than 1000 oz. The foot weighs 911.458 oz. troy, or .5274 oz per cubic inch. The cubical foot equals 2200 *cylindrical*, 3300 *spherical*, or 6600 *conical* inches. A cylindrical foot of

for 1 ton, or 2240 lbs. of freight, will weigh 5987½ lbs. *avoirdupois*. This allows nearly three times the weight of the freight in water, to the ton.

There are two methods used in reckoning tonnage, the carpenters', and the government rule.

The following is the government rule for measuring *single or double-decked vessels*:

"If the vessel be double-decked, take the length thereof, from the fore part of the main stem, to the after part of the stern post, above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel; and then deduct from the length three-fifths of the breadth, multiply the remainder by the breadth, and the product by the depth, and divide this product by

water weighs 49.1 lbs., *avoirdupois*; a cylindrical inch, .02642, and a cubic inch, .03617 lbs., *avoirdupois*.

Seawater weighs 1.03 times distilled water, which is the standard of weight: hence, 1 cubic foot of seawater weighs 1030 oz., *avoirdupois*.

Nineteen cubic inches distilled water, temperature 50° Fahr., weigh 10 oz. troy, according to act of parliament, 1825.

It may be observed here, that the *standard of liquid measure* in the United States, is the *wine gallon*, containing 231 inches, equal to 8.339 lbs *avoirdupois*, or 58372.1754 grains distilled water.

The English *imperial standard* gallon is 10 lbs., *avoirdupois*, distilled water, at 62° Fahr., and 30 inches barometer: and is about equal to 277.274 inches. It is about equal to one and one-fifth, or 1.2 gall., wine measure of the U. States.

In Great Britain, the imperial bushel weighs 80 lbs., *avoirdupois*, distilled water, at 62° Fahr., and 30 in. of the barometer. It is a vertical cylinder, 18,789 inches in diameter, and 8 inches deep; and contains 2218.192 cubic inches. This standard is adopted in New York.

The *United States standard of dry measure* is the Winchester bushel, containing 77.627413 lbs., *avoirdupois*, distilled water, maximum density, and weighed in air at 30 in. barometer. It contains 2150.42 cubic inches nearly, although 2150.2 is more used.

In Connecticut, 2198 cubic inches make a bushel. The measure used, varies in the different states: hence, the propriety of knowing the standard of the state in which the calculation is made. It should be the same in all of the states, as is our currency.

95, the quotient whereof shall be deemed the true contents or tonnage of such ship or vessel: and if such ship or vessel be single-decked, take the length and breadth, as above directed, deduct from the said length three-fifths of the breadth, and take the depth from the under side of the deck plank, to the ceiling of the hold; then multiply and divide as aforesaid, and the quotient shall be deemed the tonnage.\*

The foregoing rule may be used to get the dimensions; after this, the dimensions may be placed on the right of the line, to find the entire number of feet in the boat, and 95 on the left. The proportion will be, as 95 to the continued product of these dimensions, so will 1 ton, which the 95 feet equals, be to the whole number of tons in the vessel.

What is the government tonnage of a single-decked vessel, 115 ft. keel, 25 ft. beam, and 10 ft. hold? \*

Here,  $\frac{3}{5}$  of the width, which is 25 feet, equals 15 feet, to be subtracted from the length. This leaves the length 100 feet. The dimensions are placed on the right, and 95 on the left; thus,

In this instance, the answer is  $263\frac{1}{5}$  tons. In carpenters' measure, the  $\frac{3}{5}$  of the width would not be taken off, but would be calculated, thus,

19—95	100
	25—5
	10
	5000
	263 $\frac{1}{5}$

Here, the answer is  $302\frac{1}{5}$  tons.

19—95	115
	25—5
	10
	5750
	302 $\frac{1}{5}$

How many tons in a double-decked vessel, 268 ft. long, 30 ft. wide, and 15 ft. deep?

Here  $\frac{3}{5}$  of the width, 18 ft., are subtracted from the

\* The keel of a vessel is the main bottom in length: the beam is the greatest width from side to side of the hull; and the hold, the depth from the main deck to the bottom of the hull.

length, leaving 250 ft. long. The depth being  $\frac{1}{4}$  of the width, is 15 ft.: we state accordingly,

19— $\phi$	250
	80
1 $\phi$ —3	
	22500
	1184 $\frac{1}{4}$

From this calculation, the tonnage is 1184 $\frac{1}{4}$ .

What is the tonnage, according to carpenters' measure, of a steamboat, 300 ft. keel, 10 ft. hold, and 38 ft. beam?

We here place all of the dimensions on the right, and divide by 95; thus,

1 $\phi$ — $\phi$	300
	2 $\phi$ —2
	1 $\phi$ —2
Tons.	1200

The factor, 5, is contained in 10 twice, and in 95, nineteen times: 19 into 38, twice, and  $2 \times 2 \times 300 = 1200$  tons, the answer. Hence,

#### TO ASCERTAIN TONNAGE.

*Ascertain first the dimensions of the vessel, according to government, or carpenters' rule: place the dimensions on the right of the line, and 95 on the left: the answer will be the number of tons.*

*To ascertain any given dimension, when two of the dimensions and the tonnage are given, place the tonnage and 95 on the right, and the two given dimensions in feet, on the left: the answer will be the required dimension in feet.*

## SUPERFICIAL GEOMETRY.

### VARIETY OF FIGURES, DEFINITIONS, ETC., ETC.

A line is the shortest possible distance between two points. A line is supposed to be straight; when curved, the word, curved, is mentioned.

Two straight lines are parallel, when equally dis-

tant at all parts; and when they cannot come together, however far extended or produced.\*

The point of intersection between two lines, is called an *angle*.†

An angle of 90 degrees, or  $\frac{1}{4}$  of the circumference of a circle, is called a *right angle*: an angle of less than 90 degrees, is called an *acute*, or sharp angle.

An angle of more than 90 degrees, is called an *obtuse*, or blount angle.

A figure with four equal angles, and four equal sides, is called a *square*.‡

A figure with four equal angles, and unequal sides, is called a *rectangle*.||

A figure with four parallel unequal sides, and two acute, and two obtuse angles, is called a *parallelogram*,§ or *rhomboid*.\*\*

A figure with four equal sides, and two obtuse and two acute angles, is called a *rhomboid*; sometimes, a *lozenge*.††

\*The word *produced*, in this instance, bears its literal signification, *lead out*, from the Latin, *pro* and *duco*.

† Angle is a French word, which is from the Latin, *angulus*, a corner.

‡ Square is derived from the French, *quarre*, which is originally from the Latin, *quatuor*, four.

|| Rectangle is from the Latin, *rectus*, right, and *angulus*, an angle.

§ Parallelogram is derived from the Greek, *παράλληλος*, or *parallelos*, opposite the one to the other, and *γραμμή*, or *gramma*, a character or figure.

\*\* Rhomboid is from the Greek, *ρομβος*, or *rombos*, a *rhomb*, and *εἶδος*, or *eidos*, form. Rhomb is from the Latin *rhombus*, a *whirl*, or *constantly varying square*: in fabulous history, a varying or rolling instrument by which witches were said to bring down the moon from heaven.

†† Lozenge is from the Gr. *λοξός* or *loxos*, oblique, and *γωνία* or *gonia*, a corner

A figure with four unequal angles, and two parallel sides, is called a *trapezoid*.\*

A figure having three or more equal sides, is called a *polygon*.†

The lowest polygon, that of three sides, is called a *triangle*;‡ that of four sides, a *quadrilateral*; of five sides, a *pentagon*; of six sides, a *hexagon*; of seven sides, a *heptagon*; of eight sides, an *octagon*; of nine, a *nonagon*; of ten, a *decagon*; of eleven, an *undecagon*; and of twelve, a *dodecagon*.

There are four principal triangles: the *equilateral*, the *isosceles*, the *scaline*, and the *rectangle* triangle.

A triangle which has three sides is called *equilateral*:|| one which has two of its sides equal, is called *isosceles*:§ one which has three unequal sides, is called *scaline*:¶ and that which has a *right angle*, is called a *right angled*, or *rectangle* triangle.

The longer arm of the right-angled triangle is called the *base*, from the Lat. *basis*, the *bottom* or *foundation*: the shorter arm is called the *side*; and the side opposite the right angle, the *hypotenuse*, from the participle of

\* Trapezoid is from the Gr. *τραπέζιον*, or *trapezion*, a small table, and *ως*, or *eidos*, form.

† Polygon is from the Gr. *πολυ*, many, and *γωνια*, an angle.

‡ Triangle is from the Lat. *triangulum*, from *tres*, three, and *angulus*, a corner: hence, a figure with three angles.

Quadrilateral is from the Lat. *quatuor*, four, and *latus*, side; having four sides: pentagon, from the Gr. *πεντε*, or *pente*, five, and *γωνια*, or *gonia*, a corner. Hexagon, heptagon, octagon, nonagon, decagon, undecagon, and dodecagon are compounded by prefixing to the *γωνια*, or *gonia*, *εξ*, or *ex*, six; *επτα*, or *epta*, seven; *οκτω*, or *okto*, eight; Lat. *nonus*, nine; *duodeca*, or *dodeca*, twelve, etc.

|| Equilateral is derived from the Lat. *aequus*, equal, and *lateralis*, from *latus*, a side: equal sided.

§ Isosceles is derived from the Gr. *ισος* or *isos*, equal, and *σκελος*, or *skelos*, a leg: hence it has two equal legs.

¶ Scaline is from the Gr. *σκαληνός*, or *skalenos*, that totters or hangs over to one side, obliquely.

the Greek verb *ὑποτίθω*, or *upotithō*, extending under, or from corner to corner.

Any of the sides of an equilateral and scalene triangle may be called the base. The base of an isosceles triangle is the short side.

The upper point where the two sides of an isosceles, or other triangle, meet, is called the *vertex* of the triangle; by some, the *apex*.\*

The theory of determining angles, and the lengths of the sides of triangles, constitutes the science of *Trigonometry*; and cannot be properly treated in arithmetic. We have before seen that,

*To find the contents of a square or rectangle, multiply the two sides together.*

A parallelogram is equal in contents to a square or rectangle, when its base and vertical† height are equal to the base and side of such square or rectangle

A rhomboid or lozenge is equal to a square of the same base and altitude. Hence,

*To find the contents of a parallelogram, or rhomboid, multiply the base by the vertical height.*

*To find the contents of a trapezoid, multiply half the sum of the two parallel sides, by the vertical distance between them.*

How many yards of plastering in a wall 30 feet square?

We divide by 9 feet, which make 1 square yard.

$$\begin{array}{r|l} 30 & 30 \\ 9 & 30 \\ \hline & 100 \end{array}$$

\* These two words are frequently used synonymously. Some apply apex to triangles, and vertex to cones, because of the primary signification of vertex, from the Lat. *vertex*, a point, which is from *verto*, to turn. The plural of *vertex*, is *vertices*; and of *apex*, *apices*.

† The vertical height of any quadrilateral or four-sided figure, is a line dropped from the upper plane or vertex, perpendicular to the base. Hence, the *side* of a parallelogram must not be multiplied into the base for the contents, as this is too long, but the side arising from a line dropped at right angles with the base.

How many acres are there in the road from Cincinnati to Dayton, which is 64 miles long, and 4 rods wide?

$$\begin{array}{r}
 1\ 64 \\
 1\ 8 \\
 \hline
 1\ 60\ 4 \\
 1 \\
 \hline
 \text{Ans. } 512\ \text{A.}
 \end{array}$$

Here, we ask how many rods 64 miles will make, if 1 mile make 8 furlongs, and 1 furlong make 40 rods; then, multiplying by 4 rods in width, we say, how many acres will all these rods make, if 160 rods make 1 acre? *Ans.* 512 acres.

The first question relates to a square, or equal rhomboid; the second, to a rectangle, or equal parallelogram.

The two parallel sides of a trapezoid are 40 and 60 rods, and the distance between them 80 rods: how many acres are there?

$$\begin{array}{r}
 2-160\ 80-25 \\
 \hline
 80 \\
 \hline
 \text{Ans. } 25\ \text{acres.}
 \end{array}$$

The answer is 25 acres.

#### TRIANGLES.

Every triangle is half of a square, rectangle, parallelogram, or rhomboid of similar base and altitude. To prove this, let us, on the hypotenuse, or longest side, of any given triangle, erect another triangle, with the side and base parallel and equal, each, to the side and base of the triangle. The figure formed will be a square, rectangle, parallelogram, or rhomboid, which proves the position correct. Hence,

*To find the contents of any triangle, multiply half of the base by the whole vertical height, or the whole base by half of the vertical height, as may be most convenient.*

How many acres of land in a rectangle triangle, of 240 rods base, and 120 rods side?

Here, we place down the whole base and side, and divide by 2, which is both easy and simple; while, likewise, we have the advantage of dividing by such denominate numbers as are necessary to reduce to a given denomination.

How many square feet in an isosceles triangle, of  $16\frac{1}{2}$  ft. base, and  $37\frac{1}{2}$  ft. vertical hight?

Here, 2 is thrown on the left; and we divide by the denominators, 3 and 2. Hence, the answer,  $312\frac{1}{2}$  square feet.

$$\begin{array}{r} 350-5 \\ 2 \overline{) 700-10} \\ 2 \overline{) 625} \\ 312\frac{1}{2} \end{array}$$

If a line dropped from the vertex of a scalene triangle, fall outside of the triangle, the line of the base must be produced until it meets the vertical line.

*The square\* of the hypotenuse of a right-angled triangle, is equal to the sum of the squares of the base and side.*

For example; the base of a right-angled triangle is 8, the side 6, and the hypotenuse 10. Now,  $8 \times 8 = 64$ ; and  $6 \times 6 = 36$ ; and  $36 + 64 = 100$ ; hence, the hypotenuse is  $10 \times 10 = 100$ , which is the sum of the squares of the base and side. A rectangle, whose sides are 4, 3, and, 5, shows the same equality; thus,  $4 \times 4 = 16$ ;  $3 \times 3 = 9$ ; and  $16 + 9 = 25$ : now,  $5 \times 5 = 25$ , which proves again that the sum of the squares of the two sides equals the square of the hypotenuse.

From the foregoing, it follows, that

*To find the hypotenuse of a right-angled triangle,*

\*The square of any number, is that number multiplied into itself; 49 is the square of 7. When it is desired to square a number, 2 is written over it, thus, 7<sup>2</sup>.

*extract the square root of the sum of the squares of the base and side.\**

*To find the base, when the hypotenuse and side are given, subtract the square of the side from the square of the hypotenuse; extract the square root of the remainder, and the answer will be the base.*

*To find the side, when the hypotenuse and base are given, subtract the square of the base from the square of the hypotenuse; extract the square root of the remainder, and the answer will be the required side.*

The hypotenuse of a right-angled triangle is 10 ft., and the side 6 ft.; what is the base?

The square of the hypotenuse is  $10 \times 10 = 100$ ; and of the side  $6 \times 6 = 36$ : now,  $100 - 36 = 64$ , and the square root of 64, is 8, which is the required base.

#### POLYGONS.

If lines be drawn from the angles of the polygon, to the center, it is manifest that the polygon will be divided into isosceles triangles; and if the contents of one of these be multiplied by the whole number of angles thus made, the answer will be the contents of the polygon. Hence,

*To find the contents of a regular polygon, multiply half of the diameter of the polygon, vertical to one of the sides, by half of one of the sides; and the product by the number of sides.*

*Or, Place the shortest semi-diameter\*, and one of the sides, on the right of the line, and 2 on the left.*

\* Carpenters frequently use this method of finding the length of braces, where the distance from the angle, or lower end of the post, to the extreme end of the mortice, is given.

When the base and side of a rectangle triangle are of the same length, the hypotenuse may be found by multiplying the base or side by 1.4142. This number is the square root of twice 3.141592, which is the ratio of the circumference to the circle.

† Semi-diameter means half-diameter, from the Latin *semi*, half. *Radii* is used to denote the same thing in circles.

## THE CIRCLE.\*

*Squares* are always used as the units of superficial measurement, by reason of their sides and angles coinciding, and leaving no intervening space between their limits. The unit assumed is generally a square *inch*, a square *foot*, a square *yard*, a square *mile*, etc.

A *circle* is a plain figure, bounded by a line called the *circumference*,† which is, at all parts, equally distant from a point within, called the *centre*; hence,

The *circumference* of a circle is a line drawn at all parts equally distant from the centre.

The *diameter*‡ of a circle is a straight line drawn from the opposite sides of the circumference, through the centre, dividing the circle into two equal parts.

The *periphery*|| of a circle is its circumference; the two words being used synonymously, at pleasure.

The *radius*§ of a circle is a line drawn from the

\* Circle is derived from the Latin *circus*, a round ring, or limit; or from the Gr. *κίρκος* or *kirkos*, a falcon, that, in flying, describes circles; or from the Arabic *kara*, to go round. Many individuals confound circle with circumference; whereas, while the latter merely describes the limits, the former is the space included in such limits.

† Circumference is from the Lat. *circum*, around, and *ferentia*, from *fero*, to bear.

Centre is a French word, from a Gr. noun, signifying a goad or point, which is from the root *κέντρον* or *kentro*, to prick.

‡ Diameter is from the Gr. *διά* or *dia*, through, or through the middle, and *μέτρον* or *metron*, to measure.

|| Periphery is from *περί* or *peri*, around, about, and *φέρω* or *phero*, to bear: hence, it is identical with the circumference. The word *perimeter* is sometimes used in the same sense, but improperly. It relates particularly to the measurement or extent of circumferences, from *peri* and *metron*, to measure around.

§ Radius is a Lat. word, from *radio*, to shoot beams of light, etc. The use of this term in geometry, originates from the fact, that when a large number of radii are drawn in a circle, the circle resembles the sun darting his rays in every direction from the center.

center to the circumference, or half the diameter; two or more of these lines are called *radii*.

The *perimeter*\* of a circle, or other figure, is the extent of its circumference or bounds.

The *area*† of a circle, or other figure, is the surface or space contained within the limits of the circumference or perimeter.

A circle is said to be *inscribed* ‡ in a polygon, when the line of the circumference, cuts the sides of the polygon.

A polygon is inscribed in a circle, when its angles coincide with the circumference.

A polygon is circumscribed|| about a circle, when its sides coincide with the circumference.

A *semicircle* ¶ is a half circle, described by cutting the circumference of a circle by a right line drawn through its diameter.

An *ellipse*\*\* is an oblong, circular figure, having two axes; the *minor*, a transverse, and the *major*, a longitudinal line, each drawn through the center, and on either of which, the figure may be supposed to revolve.

\* Perimeter is from the Gr. *περι* or *peri*, *around, about*, particularly around the space described from a center, and *μετρον* or *metron*, *to measure*.

† *Area* is a Latin word, which means *space within given bounds*. Dr. Webster thinks it is from the Chaldee word *ariga*, *a bed*; or from a Hebrew word which signifies *to stretch or spread*. The plural of area is *areae*; this is, however, seldom used; being substituted by the anglicised word, *areas*.

‡ Inscribe is from the Latin *inscribo*, *to write within*.

|| Circumscribe is from *circum*, *around*, and *scribo*, *to write or draw*.

¶ Semicircle is from the Lat. *semi*, *half* and *circulus*, *a circle*.

\*\* Ellipse is from the Gr. root *ελλειπω* or *elleipo*, *to pass by or reject*.

The circumference of a circle is divided\* into 360 equal parts, called *degrees*; each of these degrees, into 60 parts, called *minutes*;† and each of these minutes into 60 parts, called *seconds*. The *degree* is marked, thus ( $^{\circ}$ ); the minute, thus ( $'$ ); the second, thus ( $''$ ). Sometimes 30 degrees are said to make 1 *sign*, marked ( $s$ ); and 12 *signs*, 1 *circle*, marked ( $c$ ).

## QUADRATURE OF THE CIRCLE.

The circle, from the varied nature of its uses and application, is one of the most interesting, and yet perplexing, figures in geometry. The comparison of circles and squares; the difficulty of determining the ratio of the circumference to the diameter; the problem of ascertaining the precise area; and the difficulty of a continued application of its principles to the measurement of solid bodies, in the form of spheres, cones, etc., have, in all ages, rendered its study peculiarly interesting to mathematicians.

We shall consider, first, the relation of circumference and diameter; next, the area; and after this, apply these principles to a great variety of practical measurements.

The difficult and impossible problem of the *quadrature\* of the circle*, is the determination of the area of a circle, whose diameter is equal to that of a given square, or of a circle inscribed in a square. The investigation of this problem commenced with Archimedes, a Grecian. The first step to be taken, was evi-

\* The division of the circle into 360 parts, originated from the division of the year by the ancients, into 360 days. The 12 signs represent the 12 months.

† Minute is derived from the Lat. *minutum*, a small part: second, from *secundus*, the second, or second order of minutes.

‡ Quadrature is from the Lat. *quadratura*, squaring, from *quatuor*, four; reducing the circle to a similar area of four equal sides.

dently to ascertain the ratio of the circumference to the diameter. And here, the investigation must be commenced, by every geometer. It is impossible to give the process and reasoning used to ascertain this, in a treatise on arithmetic; we will, however, indicate the process, and avail the benefits, without further investigation. Neither the exact ratio of the circumference to the diameter, nor the exact area of a circle, can ever be ascertained; and both have long since been abandoned, as impossible, by all good mathematicians.

The method used, is to *inscribe* and *circumscribe* the circle with regular polygons; then, to increase the number of the sides of both of these, to such an extent, that they seem to merge into a common line; and although the sides can never entirely coincide, yet they so far coincide, as to give almost entire accuracy, to all practical operations. The reason why they cannot coincide is, that a *curved* and a *straight* line can never become *one line*, however small the degree of curvature. The line of the circle is always found between these two polygons; and when the sides are increased to a very large number, the human eye, assisted by the microscope, is unable to see more than one line in the three. Archimides carried the number of sides to 32768, and thus secured the ratio to seven decimal places. He obtained the ratio  $3\frac{1}{7}$  and  $3\frac{1}{4}$ , which, reduced to an improper fraction, gave  $\frac{22}{7}$ , or the circumference to the circle, as 22 to 7. These numbers may be used for all ordinary and rough purposes; but are far from being accurate, when compared with the ratio afterward obtained by Metius, a German, who carried it to 17 places of decimals, giving the ratio of 355 to 113. Van Ceulen, a Dutch mathematician, carried it yet much further, and ascertained that if the circle was 1, the circumference would be greater than 3.14159265358579823846264-

338327950288, and less than 3.14159265358579823-846264338327950289, demonstrating the coincidence of the two polygons to a fraction less than *one nonillionth*; a difference too small to be adequately conceived. The upper number represents the inscribed, and the lower, the circumscribed polygon, between which we may vainly seek the line of the circle. Later mathematicians have carried the calculation as far as 140 places of decimals.

The relation of the circumference, may, from the above, be safely set down as 3.141592. This gives the exact ratio to 5 places of decimals, leaving a fraction as small as  $\frac{1}{1000000}$ . For all practical purposes, 3.1416 may be used, changing the 5th decimal, 9, into 6, in the 4th order.

What is the circumference of a circle, whose diameter is 31 feet?

$$3.1416 \times 31 = 97.3896 \text{ Ans.}$$

Here, we multiply by the decimals, and consequently cast off four decimals in the result. Hence,

*To find the circumference of a circle, when the diameter is given, multiply the diameter by 3.1416, and cut off four places of decimals in the answer: or, for entire accuracy, multiply by 3.1415926, and cut off seven places for decimals at the right.*

It is desired to place 40 sentinels around a camp, which is 1 mile in diameter; how far will they be apart?

We reduce the mile to yards, and get the answer in yards; they will be placed over 138 yards apart.

40	1760—44
	3.1416
	138.2304

*To find the diameter of a circle, when the circumference is given, divide the circumference by 3.1416.*

*Or, multiply the circumference by 7, and divide by 22.*

*Or, multiply the circumference by .31831.*

The circumference of a circle is 1 multiplied by 3.1416; hence, the diameter is 1 divided by 3.1416, equal to .31831, which multiplied into the circumference gives the diameter.

The circumference of a lot is 500 yards, and it is desired to plant 10 trees on the line of its diameter; how far apart will they be?

$$\begin{array}{r|l} 10 \overline{) 500} & \text{The trees will be placed nearly 16} \\ \underline{31831} & \text{feet apart.} \\ 15.91550 & \end{array}$$

It will be found far more convenient to multiply by this, than to divide by the other decimal.

#### THE AREA OF THE CIRCLE.

In treating of polygons, it has been shown, that to ascertain the area, *multiply the half radius by the entire perimeter*. It has since been shown, that the circumference or perimeter of a circle includes a vast number of polygons. Hence,

*To find the area of a circle, when the circumference and diameter are given, multiply the circumference by half the radius:*

*Or, Place the circumference and diameter on the right of the line, and 4 on the left.*

A circle is 10 feet in diameter, and 31.416 ft. in circumference; what is the area?

One fourth of the diameter, equal to the semi-radius, is  $2\frac{1}{2}$  feet, and  $31.416 \times 2\frac{1}{2} = 78.540$ ; thus,

$$\begin{array}{r|l} 3.1416 & \text{The answer is 78.54 feet.} \\ 2\frac{1}{2} \overline{) 15708} & \\ \underline{78.540} & \end{array}$$

Let us suppose a square with a circle of equal diameter inscribed: the diameter of the circle is equal to the diameter of the square:  $\frac{1}{4}$  of the perimeter of a square or circle is the side of such square or circle, which multiplied into itself will produce the area denoted by the perimeter. The square of the diameter indicates the area of the square; and 3.1416, the area of the circle: now, as  $\frac{1}{4}$  the perimeter of the square is equal to the side of the square, so  $\frac{1}{4}$  the perimeter of the circle, .7854, is equal to the side of the circle. Hence, if the square of the diameter be multiplied by  $\frac{1}{4}$  of the circumference of the circle, or .7854, the result will be the contents of the circle. For this reason, geometers take the  $\frac{1}{4}$  of  $3.141592 = .7854$ , and multiply it into the square of the diameter, when the circumference is given. Hence, the circle is .7854 of a circumscribed square. That is, if the square is 1, the circle is .7854; or, if the square contains 10,000 parts, the circle contains 7,854 parts. Hence,

*To find the area of a circle, when the diameter only is given, Multiply the square\* of the diameter by .7854, and cut off four places in the answer for decimals. For greater accuracy, multiply by .785398, and cut off six places:*

*Or, Place the diameter and 11 on the right, and 14 on the left of the line.*

What is the area of a circle whose diameter is 18 inches?

Here,  $18 \times 18 \times .7854 = 254.4696$ . The answer is  $254\frac{1}{2}$  inches, nearly.

How many acres in a circular field 40 rods in diameter?

\*The square of the diameter, is the diameter multiplied into itself.

$$\begin{array}{r}
 160 \overline{) 40} \\
 \underline{160} \\
 .7854 \\
 \text{Ans. } 7.8540
 \end{array}$$

The field contains nearly 8 acres. We divide by 160, because this number of square rods equals an acre.

What is the area of a circle  $\frac{1}{2}$  of an inch in diameter?

$$\begin{array}{r}
 21 \\
 21 \\
 \underline{21} \\
 .7854 \\
 \underline{.19635}
 \end{array}$$

The answer is very nearly  $\frac{1}{16}$  of a square inch.

The cylinder of a steam engine is 15 inches in diameter; what is the area in inches and in feet?

$$\begin{array}{r}
 15 \times 15 \times .7854 = 176 \\
 .715 \text{ inches.} \\
 \hline
 \begin{array}{r}
 4 \text{---} 12 \overline{) 15-5} \\
 2 \text{---} 1 \text{---} 12 \overline{) 15-5} \\
 \hline
 7854 \text{---} 3927 \\
 \hline
 8 \overline{) 98175} \\
 \hline
 \text{Ans. } 1.22718 \text{--- in ft.}
 \end{array}
 \end{array}$$

How many acres of land in a tract 4000 rods in diameter?

$$\begin{array}{r}
 160 \overline{) 4000} \\
 \underline{160} \\
 .7854 \\
 \hline
 78540.0000
 \end{array}$$

Here, 160 rods opposite, instead of making 1 acre, as would be the case in a square tract, make in the circular tract only .7854 of an acre; in stating, we say, how many acres will  $4000 \times 4000$  rods make, if 160 rods, make .7854 of an acre? The answer is 78,540 acres. We may ask,

What will the above land cost at  $62\frac{1}{2}$  cents per acre?

We may make this among the other statements, instead of separately; thus,

In this instance, 1 acre is placed opposite .7854 of an acre, and the price which the 1 acre equals last on the right.

	4000
160	4000
1	.7854—3927
2	125
<hr/>	
	49087.500000

How many inches in a valve  $11\frac{1}{2}$  in diameter?

Although very few figures can be canceled in this question, yet there is a decided advantage in locating the fractions on the vertical line.

	8	95
4—	8	95
	1	.7854—3927
<hr/>		
Ans.	110.7536	+

To ascertain the area of a circle, when the circumference only is given, multiply the square of the circumference by the square of 7, and the product by .7854; and divide by the square of 22.

What is the area of a circle 33 ft. in circumference.

This is an easy and simple method of arriving at the result, by one statement. The answer is  $86\frac{1}{2}$  feet. See Table of Circles and Areas.

	33
	33
	227
	227
	.7854
<hr/>	
Ans.	86.59035

#### TO FIND THE CIRCUMFERENCE OF AN ELLIPSE.

Multiply the square root of half of the sum of the two axes squared, by 3.1416; the product will be the circumference.

The axes are the longest diameters from side to side, and from end to end.

#### TO FIND THE AREA OF AN ELLIPSE.

Multiply the product of the two diameters by .7854.

TO FIND THE CONTENTS OF A SQUARE INSCRIBED IN A  
CIRCLE.

We shall endeavor to prove that such a square is one half of another square circumscribed about a circle; thus,

Let us draw a square of a given side, and inscribe in it a circle; then, in the middle of the sides of the square, where the circumference of the circle cuts the square, let us locate the four angles of a square inscribed in the circle. Thus, each side of this inscribed square will be the hypotenuse of a right-angled and equal-sided triangle. Now, from the opposite angles of the inscribed square, draw two diagonals. It will now be perceived, that the inscribed square contains four equal triangles, and the circumscribed square, eight: hence, the inscribed square is equal to half of the circumscribed square.

It is manifest, that if we square the semiradius of this circle, it will equal two of these triangles; hence, twice the semiradius, will equal four of them, which are equal to the inscribed square, or half of the circumscribed square.

What is the area of a square inscribed in a circle, 10 feet in diameter?

$$\begin{array}{r} 10 \\ 2 \overline{) 10} - 5 \\ \hline 50 \text{ ft.} \end{array}$$

Here, we place the square of the diameter on the right, and 2 on the left. This is identical with the other operation, in which the semiradius would be used; thus, semiradius, 5; this squared and multiplied by 2, thus,  $5 \times 5 \times 2 = 50$  feet, the answer, as before. Thus, by the second operation, we find it troublesome to find the semiradius, then square it, and afterward multiply it; while the other process is quite simple and easy.

How many cubic feet in a log 8 feet in diameter, and 40 ft. long?

Here, the perimeter is simply multiplied by the length and divided by 2, because the inscribed is one half of the circumscribed square. We ask again,

$$\begin{array}{r} 3 \\ 3 \\ 240-2 \\ \hline 180 \text{ feet.} \end{array}$$

What will the same come to at \$20 per 100 ft.?

Here, the statements are combined, and the answer is obtained in dollars.

$$\begin{array}{r} 3 \\ 23 \\ 10040 \\ \hline 20 \\ \hline \$36 \end{array}$$

How much will a log cost, at  $33\frac{1}{3}$  cents per foot, which is 18 inches in diameter, and 80 ft. long?

In this instance, two twelves are placed on the left, because the two dimensions on the right are inches, and must be reduced to feet. The 2 is placed on the left as usual; while  $33\frac{1}{3}$ , the price of 1 foot, is placed on the right; hence,

$$\begin{array}{r} 2-1218-3 \\ 4-1218-3 \\ 280-2 \\ 3100 \\ \hline \$30.00 \end{array}$$

*To find the contents of a square inscribed in a circle, place the square of the diameter on the right, and 2 on the left.*

*To ascertain the solid contents, place the length on the right likewise.*

*To find the price of the whole, place the price last on the right, and the quantity which it equals, on the left.*

#### TO FIND THE SIDE OF A SQUARE INSCRIBED IN A CIRCLE.

By reverting to the figure just used, we find that the side of the inscribed square is the hypotenuse of a right-angled and equal-sided triangle; hence, the square root of the sum of the squares of these two sides,

or, which is the same thing, the square root of twice the square of the radius, is equal to the hypotenuse, or side of the inscribed square. This being troublesome to attain, and knowing that the smaller square is half of the larger, we extract the square root of 3.141592, which is .707106, and multiply it into the diameter of the circle for the side of the inscribed square: likewise, we extract the square root of the product of 1 divided by 3.1416, into .707106, which is .22508; this multiplied into the circumference will give the side.

How large a square can be hewn from a round stick of timber, 20 inches in diameter?

Thus,  $20 \times .7071 = 14.142$ . The side is 14 inches and a fraction.

What is the side of a square that can be sawed from a log 60 inches in circumference?

$60 \times .22508 = 13.5048$  inches, *Ans.* We may call the last decimal .2251, instead of .2250, etc. Hence,

*To find the side of a square inscribed in a circle, multiply the diameter by .7071, or the circumference by .2251.*

THE SIDE OF A SQUARE GIVEN, TO FIND THE DIAMETER OF THE CIRCUMSCRIBED CIRCLE.

By reverting to the same figure, as above, we find that the diagonal of the inscribed square, is the hypotenuse of a right-angled and equal-sided triangle, whose sides are the sides of the square. Hence, the square root of the sum of the squares of these two sides would be the diagonal, hypotenuse, or diameter. This being tedious, we find a decimal, which multiplied into the side, will give the diagonal of the square, or diameter of the circle. This decimal is 1.4142. Likewise, the side multiplied by the decimal 4.443 will give the circumference of the circumscribed circle.

The former of these numbers is the square root of twice 3.141592; and the latter, the square root of twice this number, multiplied by itself.

How large must a tree be, in diameter, to square 12 inches?

$$12 \times 1.4142 = 16.9704 \text{ inches, Ans.}$$

How large is the circumference of a tree, or circle, around a beam or square, whose sides are 20 inches? thus,

$$20 \times 4.443 = 88.86 \text{ inches, Ans.}$$

The sides of a sill must be  $12\frac{1}{2}$  inches; how large must the tree be, in diameter, from which it is sawed?

By this statement, we avoid the difficulty of using the fractions. Hence,

25	25
1.4142	7021
Ans.	17.5525 inches

*To find the diameter or circumference of a circle circumscribed about a square whose sides are given; or, to find the diameter of a tree, that will square a given size, multiply the side of the square by 1.4142 for the diameter; and by 4.443 for the circumference or girt.*

It is frequently necessary to find the side of a square, whose area is equal to that of a given circle; or the diameter or circumference of a circle whose area is equal to that of a given square. As it belongs to geometry to demonstrate the various relative proportions of figures, we shall pursue this course no further than is demanded by a common-sense view of the subject, and give a few numbers without tracing their origin.

*To find the side of a square whose area is equal to the area of a given circle, multiply the diameter of the circle by .8862, or the circumference by .2821.*

*To find the diameter or circumference of a circle, whose area is equal to the area of a given square, multiply the side of the square by 1.128 for the diameter, and by 3.545, for the circumference.*

These rules will be found useful to mechanics and other business men generally, and are given in such form as to be easily understood and used.

Wine measure  $3\frac{1}{2}$  Gallons /  
 Beer " 36 "

252

RAINEY'S IMPROVED ABACUS.

# APPLICATION OF THE CIRCLE TO CISTERNS.\*

Cisterns are large vaults made underground to hold rainwater. They are of various shapes, square, round, and conical or pyramidal. A conical cistern is round, but of different diameters at bottom and top. A pyramidal is square, and wider at bottom or top.

All that is necessary in finding the contents of cisterns, is to ascertain the number of gallons in a solid foot, and multiply the number of feet by it. This is done, by dividing the number of inches in a cubic, or cylindric foot, by the number of inches in a wine or beer gallon.

A	cubic	foot	contains	7.48	wine	gallons.
"	"	"	"	6.127	beer	"
"	cylindric	"	"	5.875	wine	"
"	"	"	"	4.812	beer	"

The numbers 5.875, and 4.812 were obtained by multiplying 1728, the number of cubic inches in a cubic foot, by .7854, to reduce them to the number of inches in a circular or cylindric foot; the product, then divided by 231 and 282, respectively, gave the numbers, as above. Thus, we see the continued application of the decimal .7854.

Were this process not pursued, and the number of gallons in a foot obtained, it would be necessary to find the number of cubic inches in a cistern, and multiply by .7854, when round, and divide by the number of inches in a gallon.

We may dispense with the use of the decimal, partially, and multiply by two other numbers in the form of a common fraction. This is done, by multiplying the decimal by some number that will terminate in ciphers, and dividing by the same. For instance:

\* The root of cistern is the Lat. *cista*, a box, whence *cisterna*, a vault for rainwater.

$7.48 \times 2 = 14.96$ . Here, 14 is the whole number, and the .9, or  $\frac{9}{10}$ , at the right of it being very nearly a unit, we carry it to 14 and call the multiplier 15. We divide this by 2, on the left of the vertical line, as well as multiply by it, to make the other number 15.

A square cistern contains 100 cubic feet; how many wine gallons does it contain?

Here  $100 \times 7.48 = 748.00$  gallons. By the other process, we place the square of the side on the right, multiply by 15, and on the left, divide by 2; thus,

It is seen here, that the answers vary only 2 gallons in 750. Hence, it is sufficiently accurate for ordinary purposes. When strict accuracy is desired, the decimal may be used. Therefore,	<table style="border-collapse: collapse; margin: 0 auto;"> <tr><td style="border-right: 1px solid black; padding: 0 5px;">10</td><td style="padding: 0 5px;">10</td></tr> <tr><td style="border-right: 1px solid black; padding: 0 5px;">15</td><td style="padding: 0 5px;">150</td></tr> <tr><td style="border-right: 1px solid black; padding: 0 5px;">2</td><td style="padding: 0 5px;">20</td></tr> <tr><td style="border-right: 1px solid black; padding: 0 5px;">15</td><td style="padding: 0 5px;">150</td></tr> <tr><td style="border-right: 1px solid black; padding: 0 5px;">Ans.</td><td style="padding: 0 5px;">750 galls.</td></tr> </table>	10	10	15	150	2	20	15	150	Ans.	750 galls.
10	10										
15	150										
2	20										
15	150										
Ans.	750 galls.										

*To find the contents of cisterns which are square, place the square of the side, and depth in feet, with 15. on the right, and 2 on the left, for wine gallons; or multiply the number of cubic feet in the cistern by 7.48. For beer gallons, place 49 on the right and 8 on the left, or multiply by 6.127. For circular cisterns, 47 on the right and 8 on the left, or multiply by 5.875, for wine gallons: and for beer gallons, 24 on the right, and 5 on the left; or multiply by 4.812.*

*For conical or pyramidal cisterns, add the areas of the two ends: multiply the two areas, and extract the square root: add this to the sum of the two areas above: place this sum and the depth on the right, and 3 on the left: place likewise, on the right and left, the numbers representing gallons, in the measure desired, and, for square or circular cisterns, as the case may be.*

It may be remarked, that wine measure, or 231 cubic inches to the gallon, is generally used as the standard in the United States.  $31\frac{1}{2}$  gallons to the barrel

How many wine gallons in a circular cistern 12 ft. in diameter and 20 feet deep?

$$\begin{array}{r}
 12 \\
 12-8 \\
 20 \\
 4-8 \overline{) 47} \\
 \hline
 \text{Ans. } 1692 \text{ galls.}
 \end{array}$$

The same answer might be obtained by suspending the 47 and 8, and placing 5.875 on the right.

How many hogheads of water in a circular cistern, 30 feet in diameter, and 21 ft. deep, beer measure? ●

$$\begin{array}{r}
 30 \\
 30-6 \\
 21-6 \overline{) 21} \\
 24 \\
 \hline
 \text{Ans. } 1440 \text{ hhd.}
 \end{array}$$

In this instance, the answer is reduced to hogheads, by placing 63, the number of gallons in a hoghead, on the left.

How many beer gallons in a circular cistern of conical shape, which is 9 ft. in diameter at the top, 7, at the bottom, and 9 ft. deep?

$$\begin{array}{l}
 9 \times 9 = 81 \text{ area of upper end.} \\
 7 \times 7 = 49 \text{ " " lower " }
 \end{array}$$

$$81 \times 49 = \sqrt{3969} = 63 + 130 = 193.$$

$$\begin{array}{r}
 193 \\
 3 \overline{) 193} \\
 5 \overline{) 24} \\
 \hline
 5 \overline{) 13896}
 \end{array}$$

Ans. 2779½ We first square each of the diameters, and add them; next, multiply together these squares, and extract the square root; we add this root, 63, to the sum of the two squares above, 130, making 193: this 193, mean area, is placed on the line, with the depth of the cistern, while 3 is placed on the left. This is equivalent to multiplying the mean area by ⅓ of the depth. Lastly, we place 24 on the right and 5 on the left, which numbers represent both the number of gallons in a solid foot, and the deduction for the quadrature of the circle. Any conical or pyramidal cistern may be measured in the same way. It is wholly unnecessary above to get

first, the circular area by multiplying by .7854; as this can be done quite as easily, by the 24 and 5, or by the number which represents the gallons and quadrature, 4.812; for *circles and squares are to each other as the squares of their diameters.*

Required the depth of a rectangular cistern, to hold 5400 gallons, which is 8 ft. wide, and 10 ft. long.

We here place the contents of the cistern on the right of the line, and the two given dimensions, with the fraction expressing the number of gallons in a foot, on the left. The fraction in this case is  $\frac{1}{2}$ , the measure being wine gallons: thus,

Were this beer gallons, we would place  $\frac{1}{3}$  on the left. If the cistern were circular, we would place on the left  $\frac{1}{2}$ , or  $\frac{2}{3}$ , as the case might be, with the square of the diameter, to find the depth, or the depth, to find the square of the diameter. As the answer, in the latter case, would be the square of the diameter, it would be necessary to extract the square root for the diameter of the cistern. Example:

What is the diameter of a circular cistern which contains 960 gallons, beer measure, the depth being 8 feet?

The result is 25: now, the square root of this, 5, is the diameter of the cistern. Hence,

$$\begin{array}{r|l}
 10 & 5400-135 \\
 8 & \\
 \hline
 15 & 2 \\
 \hline
 \text{Ans.} & 9 \text{ feet deep.}
 \end{array}$$

*When two dimensions and the contents of a cistern are given to find the other dimension, for rectangular cisterns, place the contents on the right, and the two given sides, with the fraction representing the number of gallons in a solid foot, on the left: the answer will be the required side.*

*For circular cisterns, place the contents on the right, and the square of the diameter and fraction, on*

*the left, for the depth; or the depth and fraction, for the square of the diameter; in the latter case, extract the square root, and the answer will be the required diameter.*

TABLE,

*Giving the capacity of Square and Circular Cisterns, Wells, etc., 1 foot deep, in wine and beer gallons.*

Diam., or side in feet.	Circular Cisterns.		Square Cisterns.		
	Wi. gall.	B'r gall.	Wi. galls.	B'r galls.	
3	52.875	43.2	67.5	55.125	The table of cisterns of different figures, and of different kinds of measure, is given for the purpose of convenience. The calculations are made for cisterns 1 foot deep; therefore, nothing more is necessary in ascertaining the contents of a given cistern than to find the contents of a given diameter in the table, and multiply it by the required depth. For example: it is desired to know how much a circular cistern 20 ft. in diam. and 30 ft. deep, will contain, in wine gallons. We refer to the table and find that such a cistern 1 foot deep will contain 2350 galls., and infer that 30 times this or 70500, will be the contents of the required cistern.
3½	71.96	58.8	91.875	75.031	
4	99.4	76.8	120.	98.	
4½	118.969	97.2	151.875	124.031	
5	146.875	120.	187.5	153.125	
5½	177.719	145.2	226.875	185.906	
6	211.5	172.8	270.	220.5	
6½	248.219	202.8	316.875	258.781	
7	287.875	235.2	367.5	300.125	
7½	330.469	270.	421.875	344.531	
8	376.	307.2	480.	392.	
8½	424.437	346.0	541.875	442.531	
9	475.875	388.8	607.5	496.125	
9½	530.219	433.2	676.875	552.781	
10	587.5	480.	750.	612.5	
10½	647.719	529.2	828.875	675.281	
11	710.875	580.8	907.5	741.125	
11½	776.969	634.8	991.875	810.031	
12	846.	691.2	1080.	882.	
13	992.875	811.2	1267.5	1035.125	
14	1151.5	940.8	1475.	1200.5	
15	1321.875	1080.	1687.5	1378.125	
16	1504.	1228.8	1920.	1568.	
17	1697.875	1387.2	2167.5	1770.125	
18	1903.5	1555.2	2430.	1984.5	
19	2120.875	1732.8	2707.5	2211.125	
20	2350.	1920.	3000.	2450.	
21	2590.875	2116.8	3307.5	2701.125	
22	2843.5	2323.2	3630.	2964.5	
23	3107.875	2539.2	3967.5	3240.125	
24	3384.	2764.8	4320.	3528.	
25	3671.875	3000.	4687.5	3828.125	

CYLINDERS, CONES, SPHERES, ETC.

A *cylinder*\* is a round solid body, of uniform diameter, whose two ends or bases are at right angles with the sides.

A *cone*† is a round solid body, tapering in a direct line from the periphery of the base to a point at the top, called the vertex.

A *pyramid*‡ is a square body, tapering in a direct line from the periphery of the base to the vertex.

A *frustum*|| of a cone, or pyramid, is the part remaining after a portion of the top is cut off by a plane parallel to the base.

The *convex*§ surface of a cylinder, cone, pyramid, or frustum, is the curved surface, exclusive of the ends, or bases. The *entire surface*, includes the area of the ends, or bases.

A *sphere*¶ is a solid, round body, with a curved surface, all parts of which are equally distant from the center within; and is generated by the revolution of a semicircle about its own side.

A *spheroid*\*\* is an oblong sphere, whose diameter across is less than the diameter of the opposite ends, and is formed by the revolution of an ellipse about either of its axes. The extremes of the longer diameter are called the *major axis*, and of the shorter, the

\* Cylinder is from the Gr. root *κυλλω* or *kulio*, to roll.

† Cone is from the Welsh *con*, that which shoots a point.

‡ Pyramid originates from a Gr. word whose origin is *πυρ* or *pur*, a flame, from its resemblance in shape to a blazing fire.

|| Frustum is a Lat. word, which means a broken piece.

§ Convex is from the Lat. *convexus*, to bend down on every side, as the heavens; the opposite of concave, from *cavus*, a hollow.

¶ Sphere is from the Lat. *sphæra*, a round ball.

\*\* Spheroid is from the Gr. *σφαῖρα* or *sphaira*, a round ball, and *εἶδος* or *eidos*, form; globe-like.

*minor axis*, from the Latin, *major* and *minor*, *greater* and *less*.

The *axes*\* of a body are points on which it is supposed to revolve.

The diameter of a sphere is a line drawn through the extreme opposite surfaces. The *radius*, a line drawn from the center to any part of the surface.

A *section*, as of a cone, or other body, is a portion cut off; from the Lat. root *seco*, to cut off.†

#### THE CYLINDER.

To find the convex surface of a cylinder, multiply the circumference by the length: for entire contents, add to this twice the area of the base, or the area of the two ends.

It may be observed that the convex surface of a cylinder is a rectangular figure, supposing it to be rolled out in the form of a plane. Hence, the length multiplied by the width, will give the superficial contents, as in other cases of rectangles.

What is the convex surface of a cylinder,  $18\frac{1}{2}$  in. in circumference, and 8 ft. long?

$$\begin{array}{r} 6-12\frac{1}{2}-8-2 \\ 475 \\ \hline \end{array}$$

$12\frac{1}{2}$  feet.

Having the length in feet, we divide by 12, to reduce the width to the same dimension, that the answer may be in feet.

What is the entire surface of a cylinder 30 inches in circumference, and 40 inches long?

\* *Axes* is the plural of *axis*, which is from *αξων* or *axon*, a table at Athens on which the laws were written, and which revolved on centers or axes. This word is from *αγω* or *ago*, to guide or direct movements; hence, the direction of the motion of a body.

† This is the origin of that beautiful, yet very difficult department of geometry, called *Conic Sections*, whose demonstration requires the closest and most perspicuous reasoning.

$$\begin{array}{r}
 25 \\
 25 \\
 \hline
 227 \\
 227 \\
 \hline
 .7854 \\
 2
 \end{array}$$

$$\begin{array}{r}
 12 \overline{) 20} \\
 \underline{12} \phantom{0} \\
 8 \phantom{0} \\
 \hline
 8 \frac{1}{2} \text{ feet.}
 \end{array}$$

.9939 area of both ends.

8.3333 convex area.

Ans. 9.3272 entire surface.

In the work above, the difficulty is presented of finding the diameter of a circle, then squaring it, and multiplying by .7854, for the area, and 2, for the two ends, which would make two statements necessary. Hence, to avoid this, we multiply together the square of the circumference, the square of 7, the decimal .7854, and 2, representing the two ends, and divide by the square of 22, and the denominators found in the circumference.

*To find the solid contents of a cylinder, place the square of the diameter, the length, and .7854 on the right, and such denominate numbers as may be necessary, on the left: or, instead of .7854, place 11 on the right, and 14 on the left.*

What is the solid contents of a cylinder 80 inches in diameter, and 16 feet long?

Here the two twelves are placed on the left to reduce the square of the diameter to feet, that all of the dimensions may be in feet. Four places of decimals are cut off in the answer.

$$\begin{array}{r}
 12 \overline{) 12} \phantom{00} \\
 \underline{12} \phantom{00} \\
 16 \phantom{00} \\
 \hline
 16 \\
 .7854 \\
 \hline
 \text{Ans. } 78.5400 \text{ ft.}
 \end{array}$$

How many solid feet in a cylinder  $10 \frac{1}{2}$  feet in diameter and 12 ft. long?

$$\begin{array}{r}
 2 \overline{) 21} \\
 \underline{21} \phantom{00} \\
 21 \phantom{00} - 3 \\
 \underline{21} \phantom{00} \\
 11 \phantom{00} \\
 \underline{11} \phantom{00} \\
 2079 \\
 \underline{2079} \\
 1039 \frac{1}{2}
 \end{array}$$

The 11 and 14 are used in this calculation. These are not sufficiently accurate in cases where great precision is desired.

#### CONTENTS OF BOILERS.

To find the contents of boilers, place the square of the diameter and length in feet, on the right with 47, and 8 on the left: the answer will be the contents in wine gallons. Ascertain the contents of the flues in the same way, and subtract these from the contents above.

For beer gallons, instead of 47 and 8, use 24 and 5.

Or, Multiply the length in feet, by the contents of the given diameter, as found in the subjoined table, and strike off as many figures in the answer for decimals as there are decimal places in the multiplier.

How many wine gallons in a boiler 39 inches in diameter and 40 feet long, having two flues, each 9 inches in diameter?

$$\begin{array}{r}
 4 \overline{) 3} \\
 2 \overline{) 4} 3 \\
 \underline{8} 40 - 5 \\
 \underline{8} 40 \\
 47 \\
 \underline{47} \\
 \text{Flues.} \quad 264.375
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 12} 30 - 13 \\
 4 \overline{) 12} 30 - 13 \\
 \underline{48} 0 - 5 \\
 \underline{48} 0 \\
 47 \\
 \text{Boiler.} \quad 2482.1875 \\
 \underline{264.375}
 \end{array}$$

Ans. 2217.8125 gallons.

In this example we make a separate calculation for the flues, which are  $\frac{3}{4}$  of a foot in diameter; and as there are two of them, we multiply by 2, instead of working the question twice. The answer is 2218 gallons, nearly, wine measure. The same process may be pursued for beer gallons.

The subjoined table will be found very convenient for practical men.

TABLE

*Of contents of Boilers 1 foot in length, in wine gallons, from 3 to 38 inches in diameter.*

Dia.	Cont's.	Dia.	Cont's.	Di.	Cont's.	Di.	Cont's.	Di.	Cont's.
3	.367	7½	2.293	12	5.875	21	17.992	30	36.719
3½	.499	8	2.611	13	6.895	22	19.739	31	39.207
4	.652	8½	2.947	14	7.997	23	21.582	32	41.777
4½	.826	9	3.304	15	9.179	24	23.5	33	44.421
5	1.02	9½	3.682	16	10.444	25	25.499	34	47.163
5½	1.234	10	4.079	17	11.788	26	27.666	35	49.978
6	1.469	10½	4.498	18	13.219	27	29.308	36	52.875
6½	1.724	11	4.937	19	14.719	28	31.985	37	55.853
7	2.	11½	5.396	20	16.319	29	34.311	38	59.

AIR PRESSURE.

*To find the air pressure on piston heads and other circular areas, place the square of the diameter and 165 on the right, and 14 on the left.*

This rule is based on the supposition that the pressure of air, per superficial inch, is 15 lbs., and that  $\frac{11}{14}$  is the ratio of the circle to the square: hence, instead of using the 15 and 11, separately, we make them one number, such as may be easily remembered, by multiplying them together. The pressure may be quite accurately obtained by multiplying by 59 and dividing by 5; though, for entire accuracy, it is best to use 15 and the decimal .7854, which, multiplied into the square of the diameter, will give the true result.

What is the air pressure on a piston head 10½ inches in diameter?

The answer is nearly 1300 pounds.

$$\begin{array}{r}
 2 \overline{) 21} \\
 \underline{22} \text{—} 3 \\
 2 \overline{) 14} \text{—} 165 \\
 \underline{8} \text{—} 10395 \\
 \text{Ans. } 1299\frac{3}{4} \text{ lbs.}
 \end{array}$$

What is the air pressure on a piston head 35 inches in diameter, in tons?

$$\begin{array}{r}
 2-2-100\cancel{3}5-7 \\
 2-20\cancel{3}5-5 \\
 2-141\cancel{6}5-33 \\
 \hline
 32\ 231 \\
 \hline
 \text{Ans. } 7\frac{7}{31}
 \end{array}$$

The answer is  $7\frac{7}{31}$  tons, equal to 7 tons, and  $437\frac{1}{2}$  lbs., avoirdupois.

## CONES AND PYRAMIDS.

To find the convex surface of a cone or pyramid, multiply the circumference or perimeter of the base by the slant height, and divide by 2.

The reason for dividing by 2, is that the convex surface of a cone or pyramid is just  $\frac{1}{2}$  the convex surface of a cylinder of similar base and altitude.

The circumference of a cone is 90 inches, and the slant height 10 feet; how many feet in its surface?

$$\begin{array}{r}
 4-12\cancel{9}0-3 \\
 21\cancel{0}-5 \\
 \hline
 4\ 150 \\
 \hline
 \text{Ans. } 37\frac{1}{2} \text{ ft.}
 \end{array}$$

Twelve is used on the left, to reduce the circumference to feet.

To find the solidity of a cone or pyramid, multiply the square of the diameter, the vertical height, and .7854 together, and divide by 3, and such other numbers as may be necessary to reduce the dimensions to the same denomination. Cut off four figures for decimals in the answer.

The reason for dividing the product of the dimensions by 3, is, that a solid cone is one-third of a cylinder of similar base and altitude. The same thing is true of a pyramid.

How many solid feet in a cone 10 feet in diameter, and 30 feet high?

$$\begin{array}{r}
 10 \\
 3\ 10 \\
 30 \\
 .7854 \\
 \hline
 \text{Ans. } 785.4000
 \end{array}$$

The answer is  $785\frac{1}{3}$  solid feet, nearly. We might use the 11 and 14, instead of .7854, which would give the same answer within a very small fraction.

To find the convex surface of the frustum of a cone or pyramid,

## SOLIDITY OF THE FRUSTUM OF A CONE. 203

*place the sum of the circumferences, or perimeters, of the two ends with the slant height on the right, and 2 on the left; for the entire surface, add to the result, the sum of the areas of the two ends.*

A cone, superficially measured, may be easily proved to be a triangle; hence, the necessity of dividing the product of its two sides, as a rectangle, by 2, to get the triangle. Likewise, the superface of a frustum may be divided into two triangles: hence, the necessity of the sum of the circumferences.

*To find the solidity of the frustum of a cone or pyramid, To the sum of the areas of the two ends, add the square root of the product of these areas; place this sum, with the vertical height, on the right of the line, and 3 on the left.*

Thus we obtain the mean and two end areas, which are the bases of three right cones: these three cones are equal to the solid frustum.

How many solid feet in the frustum of a cone  $4\frac{1}{2}$  ft. high, and 6 ft. in diameter at the lower, and 5 ft. at the upper end?

$$6 \times 6 = 36 \times .7854 = 28.27 \text{ area of lower base.}$$

$$5 \times 5 = 25 \times .7854 = 19.63 \text{ " " upper "}$$

$$47.9 \text{ sum of bases.}$$

$$28.27 \times 19.63 = \sqrt{554.94} = 23.55.$$

Now,  $47.9 + 23.55 = 71.45$ , and this multiplied by the height, and divided by 3, gives the answer; thus,

In the work above, we ascertained, first, the two areas, multiplied them, and extracted the square root of their product, which root was 23.55. This, added to the sum of the two areas, 47.9, gave 71.45, to be multiplied by $\frac{1}{3}$ of the height. It is almost impossible to explain such operations as this satisfactorily by arithmetic; yet their utility in practical affairs necessitates their introduction.	$\begin{array}{r} 3 \overline{) 71.45} = 1429 \\ 5 \overline{) 24} = 8 \\ \hline \text{Ans. } 114.82 \end{array}$
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## THE SPHERE OR GLOBE.

To find the surface of a sphere or globe, Multiply the circumference by the diameter :

Or, Multiply the square of the diameter by 3.1416 :

Or, Multiply the square of the circumference by .3183.

The decimal .3183 is obtained by dividing 1 by 3.14159.

What is the surface of a sphere, whose diameter is 3 feet? thus,

$3 \times 3 = 9$ , the square of the diameter; and this  $9 \times 3.1416 = 28.2744$ , or 28 superficial feet, the answer.

To find the solidity of a sphere,

Multiply the cube\* of the diameter by the decimal 5236 :

Or, Multiply the square of the diameter, .7854,  $\frac{1}{4}$  of the diameter, and 4 together :

Or, Multiply the square of the diameter by 3.1416, and the product by  $\frac{1}{4}$  of the diameter.

The decimal .5236 here used, is one-sixth of 3.1416. Indeed, all of the decimals used as multipliers in anything pertaining to circular figures, can be traced back to the number representing the ratio of the circumference of a circle to its diameter.

A globe is 10 inches in diameter; how many solid inches does it contain?

$$10 \times 10 \times 10 = 1000 \times .5236 =$$

523.6 cubic inches, Ans.; or,

$$10 \times 10 \times .7854 \times 1\frac{3}{4} \times 4 = 523.6; \text{ or,}$$

10	
10	
3.1416	
35	
Ans.	523.6000

Any of the processes above may be used with entire safety.

\* The cube of a number is that number multiplied into itself twice; the cube of 4 is 64.

A globe is 30 inches in diameter; how many solid feet does it contain?

It is frequently quite convenient to state on the line, where reduction is to be performed, or any fractional numbers used. The question may be stated thus,

$$\begin{array}{r|l}
 4-\cancel{12}30 & \\
 4-\cancel{12}30 & \\
 12 & 3.1416 \\
 \hline
 & 5 \\
 \hline
 \text{Ans.} & 8.18125
 \end{array}$$

The two dimensions in the square are reduced to feet, while the third dimension, in the cube, is still inches, and must be reduced by 12.

$$\begin{array}{r|l}
 25 & \\
 25 & \\
 12 & 3.1416 \\
 \hline
 & 5 \\
 \hline
 \end{array}$$

It may be remarked, that when a cylinder is circumscribed about a sphere, whose length is equal to its own diameter, or that of the sphere, the relation of the surface of the sphere to the entire surface of the cylinder is as 2 to 3; and that the relation of their solidities is the same. Hence, an easy method of finding the solidity of a sphere, is to take two-thirds the solidity of a cylinder, of diameter and length equal to the diameter of the sphere.

#### THE SPHEROID.

*To find the solidity of a spheroid,*

*Multiply the square of the shorter or minor axis by the longer or major axis, and the product by .5236.*

A spheroid is 20 by 30 inches; what is its solidity?  
 $20 \times 20 \times 30 \times .5236 = 6283.2$  cubic inches. Ans.

*Lines are to each other as their linear extent:*

*Areas are to each other as their squares: and*

*Solidities are to each other as their cubes. Therefore,*

*To ascertain the weight of a globe, when the weight of a globe of similar material is given, Place the cube of the diameter of the globe whose weight is required, on the right, and the cube of the globe whose diameter is given, on the left, and the weight of the given globe, on the right.*

The solidity of a globe, whose diameter is one inch, is .52359, which, for practical purposes, is called .5236.

A globe of wrought iron, 1 foot in diameter, weighs 254.8 lbs., and of cast iron, 242. The weight of bar iron being 1, the weight of cast iron is .95, of steel, 1.02, copper, 1.16, brass, 1.09, and lead, 1.48.

A cubic foot of rolled iron weighs 486.65 lbs., *avoirdupois*; a cylindric foot, 382.2 lbs. Hence, a cubic inch weighs .28166 of a lb.; a cylindric inch, .22116; now, taking two-thirds of the latter, shows that a spherical inch of rolled iron weighs .14744, and of cast iron, .14006 of a lb. A cubic inch of cast iron weighs .26757 of a lb.; hence, 3.84 cubic inches of cast iron weigh 1.02 lb.; 3.84 cubic inches are generally allowed for 1 lb.

If a cast iron globe, 12 inches in diameter, weighs 242 lbs., how much will a globe of the same metal weigh, which is 15 inches in diameter?

$$\begin{array}{r}
 4 \text{ --- } 12 \mid 15 \text{ --- } 5 \\
 4 \text{ --- } 12 \mid 15 \text{ --- } 5 \\
 2 \text{ --- } 4 \text{ --- } 12 \mid 15 \text{ --- } 5 \\
 \hline
 \phantom{2 \text{ --- } 4 \text{ --- } 12 \mid } 242 \text{ --- } 121 \\
 \hline
 \text{Ans. } 472\frac{2}{3}
 \end{array}$$

Here, the cube of the diameter is placed on the line in inches, in each case; and as the cube of 12 equals 242, the cube of 15 must equal 473, etc.

#### GAUGING.\*

Gauging is the measurement of casks or barrels; and is subject to no specific rules, from the fact that a cask is not identical with any regular geometrical figure; hence, no certain directions can be given to measure all of the various shapes which casks assume. They are generally considered the two equal frustums of a cone, with greater or less lateral curvature. It is not necessary to enter into an investigation of the principles on which we found the following

\* Gauge is from the French, *jauge*, a measuring rod.

## DIRECTIONS FOR GAUGING CASKS.

*Place the sum of twice the square of the bulge diameter and once the square of the head diameter, with the length, on the right; and, on the left, 882 for wine gallons, and 1077 for beer gallons: Or, Multiply the square of the mean diameter by the length, and the product by .0034 for wine, and .0028 for beer gallons.*

To ascertain the mean diameter between the bung and the head, where the stave is greatly curved, add to the head diameter .7 of the difference between the head and bung diameters; when moderately curved, .55; and when very slightly curved, .5.

How many wine gallons in a cask 49 inches long, 30 inches bung, and 21 inches head diameter? thus,

$  \begin{array}{r}  30 \times 30 \times 2 = 1800 \\  21 \times 21 = 441 \\  \hline  2241  \end{array}  $	$  \begin{array}{r}  18-126-882 \quad 2241-747 \\  \hline  18-126-882 \quad 18-7 \\  \hline  \phantom{18-126-882} \quad 6747 \\  \hline  \text{Ans. } 124\frac{1}{2} \text{ galls.}  \end{array}  $
---	---

The measurement of casks by calculation is of but little utility, as it is now mostly done by a rod, with computations already made, in tabular form.

## MECHANICAL POWERS.

The remarks on the mechanical powers will be very limited, as this subject belongs legitimately to natural philosophy. Yet we may give such general directions as will enable the student in philosophy to make his calculations with greater facility, than by the old method.

The mechanical powers are six: the *lever*, the *inclined plane*, the *wheel and axle*, the *pulley*, the *screw*, and the *wedge*.

Several of these are, however, the same powers, which receive their name from the nature of their application; as there are, strictly speaking, but two powers, the *lever* and *inclined plane*. The *wheel* and *axle*, and *pulley* are revolving levers; the *screw*, a revolving inclined plane; and the *wedge*, a compound inclined plane.

The *fulcrum*\* of a lever is the point or pivot on which the lever rests: the *arm* is the distance between this rest and the power or weight.

When the two arms of a lever and the power are given, to find the weight that will equiponderate, proceed as in *Inverse Proportion*: Or, Place the length of the arm for which the weight is required, on the left, and the other arm and the given weight on the right: the answer will be the required weight.

A lever 20 feet long rests on a fulcrum 5 feet from one end; on the short end is a weight of 3000 lbs.: what weight attached to the other end will equiponderate?

$\begin{array}{r} 20 \text{ feet} \\ \hline 5 \text{ feet} \end{array}$	$\begin{array}{r} 3000 \text{ lbs.} \\ \hline 1000 \text{ lbs.} \end{array}$	This may be proven by finding the length of one of the arms.
---	--	--

If the arm of a lever 15 feet long, with 1000 lbs. attached, equiponderate 3000 lbs., how long is the arm to which the latter weight is attached?

$\begin{array}{r} 3000 \text{ lbs.} \\ \hline 1000 \text{ lbs.} \end{array}$	$\begin{array}{r} 15 \text{ feet} \\ \hline 5 \text{ feet} \end{array}$	Here, 3000 feet is the demand, 1000 the same name, while the term of answer is 15 feet. The 1000 lbs. and the 15 coöperate to produce the common effect, <i>equiponderance</i> . Hence, they are causes in proportion.
--	---	--

When a weight is sustained between two props or fulcra, proceed by *Inverse Proportion*; making the entire length of the lever the demand, the short arm the same name, when the weight on the

\* Fulcrum is a Latin word which means a *prop*, or *brace*.

*prop of the long arm is required, and vice versa, and the whole weight the term of answer.*

Two men, A and B, carry a burden on a lever 30 feet long, placed 10 feet from A; what is the weight sustained by B?

$$\begin{array}{r|l} 30 & 20 \\ \hline & 400 \\ \hline \text{Ans.} & 266\frac{2}{3} \text{ A's.} \end{array}$$

$$\begin{array}{r|l} 30 & 10 \\ \hline & 400 \\ \hline \text{Ans.} & 1.33\frac{1}{3} \text{ lbs. B's.} \end{array}$$

*The weight to the answer is inversely as the arm to the whole lever. These two results added, make 400 lbs., the whole weight.*

*The diameter of the wheel, the diameter of the axle, and the power given, to find the weight: Proceed as in Inverse Proportion, etc.*

The diameter of a wheel is 21 ft., and that of the axle 8 inches; what weight attached to the axle will balance 140 lbs. attached to the periphery of the wheel?

We here make the 8 inches  $\frac{2}{3}$  of a foot, and placing it on the left, divide likewise by 100 which reduces the answer to cwts.: hence,  $50\frac{2}{3}$  cwt. The radius of a wheel is the long arm; the radius of the axle; the short arm. The weight is the power. This question may be proven by finding the power attached to the wheel, or by finding the radius of either the wheel or axle. The ingenious pupil may experiment at pleasure.

$$\begin{array}{r|l} 21 & 3 \\ \hline 10 & 21 \\ \hline & 140-7 \\ \hline 10 & 504 \\ \hline \text{Ans.} & 50\frac{2}{3} \end{array}$$

*The length, the height, and the power of an inclined plane given, to find the weight: Make the length the demand in direct proportion; the height, the same name; and the power, the term of answer: the answer will be the weight.*

*To find the power, make the height the demand, etc.*

An inclined plane is 72 ft. long, and 8 ft. high; what weight will 781 lbs. power sustain?

$$\begin{array}{r} 9 \\ 781 \\ \hline \text{Ans. } 7029 \text{ lbs.} \end{array}$$

The demand, 72, is placed on the right. Again:

What power will sustain 7029 lbs. weight on an inclined plane 72 ft. long, and 8 ft. high?

$$\begin{array}{r} 8 \\ 7029 \\ \hline \text{Ans. } 781 \text{ lbs.} \end{array}$$

We now get the power assumed in the first question, for the answer.

*The side of a wedge, the thickness of the head, and the power given, to ascertain the force: Make the length the demand; the thickness, the same name; and the power, the term of answer.*

*The dimensions and resistance given, to find the power: Make the thickness the demand; the length the same name; and the resistance the term of answer.*

The length of a wedge is 40 inches, the head 8 inches, and the power 300 lbs. what is the force?

$$\begin{array}{r} 8 \\ 40 \\ \hline \text{Ans. } 1500 \text{ lbs.} \end{array}$$

This may be proven by finding the power, the force being the resistance; thus,

$$\begin{array}{r} 40 \\ 1500 \\ \hline \text{Ans. } 300 \text{ lbs.} \end{array}$$

These statements are made by direct proportion.

*The distance between the threads of a screw, the length of lever, and power given, to ascertain the weight: Make the circumference whose radius is the lever, the demand; the distance between the threads, the same name; and the power the term of answer.*

*The weight given, to ascertain the power: Make the distance between the threads, the demand; the circumference, as above, the same name; and the power, the term of answer.*

The distance, parallel to the center, of a screw between its threads, is  $2\frac{1}{4}$  inches; the length of the lever, 56 $\frac{1}{4}$  inches, and the power, 9 tons; what pressure will it give?

Here, 355 is the circumference of  
twice the radius,  $56\frac{1}{2}$ .

5	355
2	
9	
Ans.	1278 tons.

Let us now find the power, the pressure, 1278 tons,  
being ascertained; thus,

We merely reverse the supposition and demand, after finding weight, force, pressure, etc., to find the power.	355	5	
		2	1278
		Ans.	9 tons.

We trust enough has been said to render the appli-  
cation of arithmetic to philosophy, plain and simple, so  
far as the mechanical powers are concerned.

## SQUARE ROOT.

The extraction of the square root, and all the other  
roots, depends on principles which it is extremely dif-  
ficult to explain satisfactorily, in arithmetic. The  
roots belong properly to Algebra; and the only ex-  
cuse for the notice of square root here, is, that we  
very frequently require it in practical affairs. Cube  
root, to the contrary, is very seldom needed, except by  
scientific men, such as have thoroughly studied all of  
the principles of algebra. I have never, in the course  
of my life, found occasion for extracting the cube root  
once, for practical purposes. Hence, the propriety of  
excluding it from this treatise; as likewise all of those  
subdivisions of numbers whose explanation depends  
on algebraic principles; such as the Positions, Alliga-  
tion, the Progressions, Permutation, etc., etc.; none of  
which offer any reward for the arduous labor lost in the  
impossible task of their attainment in arithmetic. If

one-half the time devoted to these principles in their arbitrary form in arithmetic, were devoted to the study of algebra, the pupil would learn a great portion of that beautiful science, and thus secure the *only key to the principles involved in these rules.*

The sign  $\sqrt{\phantom{x}}$  placed before a number, indicates that the square root is to be extracted. By placing 3, 4, 5, etc., over it, we understand that the cube, fourth, or fifth root is to be extracted; thus,

$$\sqrt[3]{16}=4; \text{ and } 4 \times 3 = \sqrt[4]{144}.$$

$$\sqrt[5]{125}=5; \text{ and } 5 \times 2 = \sqrt[5]{1000}.$$

#### SUMMARY OF DIRECTIONS.

I. *Separate the number into columns or periods of two figures each, by placing a period (.) over the unit's figure, and over every second figure from this to the left, and in decimals, over every second figure toward the right.*

II. *Find the greatest square number in the first period to the left, and place the root, or an equal factor of such square number, for the quotient, at the right of the whole number: subtract the square of this root, or quotient figure, from the first period, and to the right of the remainder bring down the two figures of the next period, for a new dividend.*

III. *Double the root or quotient figure obtained, and place it to the left of the new dividend, for a new partial divisor: ascertain how many times it is contained in the new dividend, exclusive of the right-hand figure of such dividend, and place the quotient to the right of the first quotient or root: place this quotient, or second figure of the root, likewise to the right of the partial divisor, which was used in obtaining it: multiply the whole number thus found as the divisor, by the figure thus appended, or the last figure in the root: subtract the product, and bring down the next period, for a new dividend.*

IV. *Proceed with this period as the one preceding, and thus continue the operation, until the roots of all the periods are extracted.*

V. *If there be a remainder after all the periods are thus used, two ciphers may be added at a time, and the operation continued to any desired number of decimal places.*

VI. *The work is correct, if the root multiplied by itself, gives a product equal to the original number.*

What is the square root of 729?

It this example, we make 7 the first, and 29 the second period: 2 squared, equal to 4, makes the largest number that can be extracted from 7; for 3 squared, would give 9, a number too large. We place this 2 to the right, subtract its square, leaving 3, and bring down the 29: we now square the 2, placing the square 4, on the left of the new dividend, 329; we divide the 4 into the first two figures of the dividend, 32, and find that it would go eight times, and conclude that 8 must be placed to the right of the divisor, 4; but, when we multiply the 48 thus found by the root 8, we find that it makes 384, a number that cannot be subtracted from 329. Hence, we conclude that the partial divisor, 4, must go into 32 only seven times, making allowance for the one that will be carried from the 9 which is rejected, and place the 7 in in the root, and also at the right of the divisor 4, making 47. Now, this 47, multiplied by the root, 7, makes 329, which subtracted leaves nothing. We, therefore, conclude that the root is 27, and prove it by finding that  $27 \times 27 = 729$ . Again:

$$\begin{array}{r} 2)729(37 \\ \underline{4} \\ 47)329 \\ \underline{329} \\ 0 \end{array}$$

What is the side of a square field that contains 42025 square roods?

We first divide this number into periods; thus,

The first root is 2, and its product, 4, subtracted leaves nothing. 20 being the next period, we know that 4, which is the root doubled, would be contained 5 times; but, as 45 multiplied by 5 could not be subtracted from 20, we say that this period gives no root figure, and supply its place by a cipher in the root, and also a cipher at the right of the partial divisor, 4. Now, this partial divisor,

$$\begin{array}{r} 2)42025(205 \\ \underline{4} \\ 405)2025 \\ \underline{2025} \\ 0 \end{array}$$

40, is contained in the 202, five times: hence, we place 5 in the root, and 5 at the right of 40, and multiply the divisor thus found by the 5, making 2025. Hence, the root is 205. This multiplied by itself gives the original number.

In decimals appended to a whole number, the extraction is effected as in whole numbers. The only thing to be observed, is to place the decimal point after the last root figure in the whole numbers, and all of the remaining figures will be decimals.

*To ascertain the square root of a vulgar fraction, reduce the fraction to its lowest term, and extract the roots of the numerator and denominator, separately.*

What is the square root of  $\frac{81}{144}$ ?

$\frac{81}{144} = \frac{9}{16}$ ; now,  $\frac{\sqrt{81}}{\sqrt{144}} = \frac{3}{4}$ ; or, get the roots of the fraction, and reduce these to the lowest term; thus,  $\frac{\sqrt{81}}{\sqrt{144}} = \frac{9}{12}$ ; and  $\frac{9}{12} = \frac{3}{4}$ . *Ans.*

A number whose root cannot be exactly ascertained, is called a *surd*.

## CURRENCY.

*Value of foreign Gold and Silver Coins, according to Custom-House usage.*

Guinea, English, gold,	\$5.00	Leghorn Dollar, silver,	\$0.90
Crown, " silver,	1.12	Soudo, of Malta, "	.40
Shilling, " "	.23	Doubleon, of Mexico, gold,	15.00
Bank token, English, silver,	.25	Livre, of Neuchâtel, silver,	.20½
Florin, of Basle, silver,	.41	Half Joe, Portugal, gold,	8.53
Moidore, Brazil, gold,	4.80	Florin, Prussia, silver,	.22½
Livre, of Catalonia, silver,	.53½	Imperial, Russia, gold,	7.83
Florence Livre, silver,	.15	Rix Dollar, Rhenish, silver,	.60½
Louis d'or, French, gold,	4.56	" " Saxony, "	.60
Crown, " silver,	1.06	Pistole, Spanish, gold,	3.97
40 Francs, " gold,	7.66	Rial, " silver,	.12½
5 Francs, " silver,	.93	Cross Pistareen, "	.10
Geneva Livre, silver,	.21	Other Pistareens, "	.18
40 Thalers, German, gold,	7.80	Swiss Livre, "	.27
10 Paolas, Italy, silver,	.97	Crown, of Tuscany, silver,	1.05
Jamaica Pound, nominal,	3.00	Piaster, Turkish, "	.05

# FOREIGN COINS AND MONIES OF ACCOUNT. 275

## Monies of Account and Coins, made current by act of Congress.\*

<b>Found Sterling, G. Britain,</b> \$4.84 <b>Do., Canada and N. Soo.,</b> 4.00 <b>Do., N. Brn. and N. Found.,</b> 4.00 <b>Franco, of France and Belgium,</b> .186 <b>Livre Tourneis, France,</b> .185 <b>Florin, Netherlands,</b> .40 <b>Do., Southern Ger. States,</b> .40 <b>Guilder, of Netherlands,</b> .40 <b>Real Vellen, Spain,</b> .05 <b>Do., Plate, " "</b> .10 <b>Milree, of Portugal,</b> 1.12 <b>Milree, of Azores,</b> .83½ <b>Marc Banco of Hamburg,</b> .35 <b>Thaler, or Rix Dollar, Prussia,</b> and Nor. Ger. States, .40	<b>Rix Dollar of Bremen,</b> \$0.78½ <b>Specie Dollar of Denmark,</b> 1.05 <b>Do., Sweden and Norway,</b> 1.06 <b>Rouble, Russia, silver,</b> .75 <b>Florin, of Austria,</b> .485 <b>Lira or Lombardo, Venetian</b> kingdom, .16 <b>Lira, of Tuscany,</b> .16 <b>Lira, of Sardinia,</b> .186 <b>Ducat, Naples,</b> .80 <b>Ounce, of Sicily,</b> 2.40 <b>Livres, Leghorn,</b> .16 <b>Tael, of China,</b> 1.43 <b>Rupee, Company,</b> .445 <b>Pagoda, India,</b> 1.84
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## Foreign Monies of Account, giving the value of the unit, according to custom, in dollars and cents.†

<b>Brazil.</b> —1000 Rees — 1 Milree — in Federal money to	\$0.823
The silver coin, 1200 Rees —	.994
<b>Bremen.</b> —5 Schwarcs — 1 Grote; 72 Grotes — 1 Rix Dollar, silver,	.787
<b>Belgium.</b> —100 cents — 1 Guilder or Florin; 1 Guilder, (silver),	.40
<b>Bencoolen.</b> —8 Satellers — 1 Soocoo; 4 Soocoos — 1 dollar or rial,	1.10
<b>Austria.</b> —60 Kreutzers — 1 Florin; 1 Florin, silver —	.485
<b>British India.</b> —12 Pice — 1 Anna; 16 Annas — 1 Co Rupee, silver,	.445
In Bengal, Madras, and Bombay, the current silver Rupee —	.444
<b>Buenos Ayres.</b> —8 Rials — 1 dollar, common currency, (fluctuating),	.93
<b>Canton.</b> —10 Cash — 1 Candarine; 10 Candarines — 1 Mace; 10 Mace — 1 Tael,	1.48
(The Cash, composed of Copper and lead, is said to be the only money coined by the Chinese.)	
<b>Cape of Good Hope.</b> —6 Stivers — 1 Schilling; 8 sch. — 1 Rix dollar,	.313
<b>Ceylon.</b> —4 Pice — 1 Fanam; 12 Fanams — 1 Rix dol.,	.40
<b>Cuba.</b> —8 Rials, plate, — 1 dollar; 1 dollar,	1.00
<b>Columbia, Ecuador, Venezuela, and New Grenada.</b> —8 Rials — 1 dollar; 1 dollar, fluctuating,	1.00

\* Monies of Account are not represented by coin, and are used for facility in reckoning, only, as mills in our country. To verify the tables above, see *Laws of the United States*.

† See *Encyclopædia Britannica*, and *McCulloch's Commercial Dictionary*.

<i>Chili</i> .—8 Rials —1 dollar; 1 dollar, silver, —	\$1.00
<i>Denmark</i> .—12 Pfenings —1 skilling; 16 Sk. —1 Marc; 6 Marcs —1 Riggsbank, or 1 Rix dollar, silver,.....	.52
<i>Egypt</i> .—3 Aspers —1 Para; 40 Paras —1 Piaster, sil.,	.048
<i>Hamburg</i> .—12 Pfenings —1 Schilling or Sol; 16 Schil. —1 Marc Lubs;* 3 Marcs —1 Rix dollar, Current	
Marc, silver,.....	.28
Marc Banco, .....	.35
<i>Holland</i> .—100 Cents —1 Florin, or Guilder: 1 Florin, silver, .....	.40
<i>Greece</i> .—100 Lepta —1 Drachmé; 1 drachmé, silver,..	.166
<i>Great Britain and France</i> .—See tables above.	
<i>Japan</i> .—10 Candarines —1 Mace; 10 Mace —1 Tael,	.75
<i>Malta</i> .—20 Grani† —1 Taro; 12 Tari —1 Scudo; 2½ Scudi —1 Pezza,.....	1.00
<i>Java</i> .—100 Cents —1 Florin; 1 Florin, as in Nether- lands,.....	.40
<i>Mauritius</i> .—In accounts of state, 100 Cts. —1 dol. —	.968
<i>Manilla</i> .—34 Maravedies —1 Rial; 8 Rials —1 dollar, Spanish, .....	1.00
<i>Milan</i> .—12 Denari —1 soldo; 20 Soldi —1 Lira,.....	.20
<i>Mexico</i> .—8 Rials —1 dollar, 1 dollar, .....	1.00
<i>Monte Video</i> .—100 Centesimi —1 Rial; 8 Rial —1 dol.,	.833
<i>Naples</i> .—10 Grani —1 Carlino; 10 Carlini —1 Ducat, silver, .....	.80
<i>Netherlands</i> .—Throughout the whole kingdom, ac- counts are kept in Florins or Guilders, and cents, as per law of 1815. See Holland.	
<i>New South Wales</i> .—Accounts are kept in Sterling Money, only.	
<i>Norway</i> .—120 Skillings —1 Rix dollar, silver,.....	1.06
<i>Papal States</i> .—10 Bajocchi —1 Paolo; 10 Paoli —1 Scudo or Crown, .....	1.00
<i>Peru</i> .—8 Rials —1 dollar, silver, .....	1.00
<i>Portugal</i> .—400 Rees —1 Cruzado; § 1000 Rees —1 Mil- ree or Crown,.....	1.12

\* Lubs indicates that it is the money of the city Lubec; the common coin is the marc currency; the marc banco represents the certificates of deposit of bullion, jewelry, etc., in the bank of Hamburg. Invoices and accounts are frequently made in Flemish pounds, shillings, and pence, which are subdivided as sterling money. The Flemish pound is equal to 7½ marcs banco.

† Grani is the plural of grano; tari, plural of taro; sudi, of scudo; lire, of lira; pezza of pezza; soldi, of soldo; carlini, of carlino; bajocchi, of bajocco; and paoli, of paolo.

‡ Norway has no gold coin of her own.

§ Cruzado is plural of cruzado; groschen, of grosch; centesimi of centesimo; lire piccole, of lira piccola; soldi di pezza, of soldo di pezza.

<i>Prussia</i> .—12 Pfennings —1 Grosch, silver; 30 Groschen —1 Thaler, or dollar, .....	\$0.69
<i>Russia</i> .—100 Copecks —1 Rouble, silver, .....	.78
Accounts were kept in paper Roubles previous to 1840, $3\frac{1}{2}$ of which were equal to 1 silver Rouble.	
<i>Sardinia</i> .—100 Centesimi —1 Rira; 1 Lira —1 Franc, French, .....	.186
<i>Sweden</i> .—12 Rundstycks —1 Skilling; 48 Skilling —1 Rix dollar, specie, .....	1.06
<i>Sicily</i> .—20 Grahi —1 Taro; 30 Tari —1 Oncia, gold, .....	2.40
<i>Spain</i> .—2 Maravedies —1 Quinto; 16 Quintos —1 Rial of old plate;* 20 Rials vellon —1 Span. dollar, .....	1.00
<i>St. Domingo</i> .—100 Centimes —1 dollar: 1 dollar, .....	$.33\frac{1}{2}$
<i>Tuscany</i> .—12 Denari di Pezza —1 Soldo di Pezza; 2 Soldi di Pezza —1 Pezza of 8 Rials; 1 Pezza, silver, .....	.90
<i>Turkey</i> .—3 Aspers —1 Para; 40 Paras —1 Piaster, varying, .....	.05
<i>Venice</i> .—100 centesimi —1 Lira; 1 Lira —1 Franc, Fr., Accounts were once kept in ducats, lire, etc. 12 Denari —1 Soldo; 20 Soldi —1 Lira Piccola; $6\frac{1}{2}$ Lire piccole —1 Ducat current; 8 Lire pic. —1 Ducat effective; the Lira piccola is worth, .....	.186
<i>West Indies, British</i> .—Pounds, shillings, pence, etc., as in England; the value varies in the different islands, and is in all of them below that of England.	.096

*Jewish or Scripture, Standard Weights and Measures.†*

**WEIGHTS OF MONEY.**—60 Shekels —1 Maneh; 50 Maneh —1 Talent; or 113 lbs., 10 oz., 1 dwt., 10 grs., Troy.

**LONG MEASURE.**—4 Digits —1 Palm; 3 Palms —1 Span; 2 Spans —1 Cubit; 4 Cubits —1 Fathom; 2 Fathoms —1 Arabian Pole; 10 Poles —1 Schœnus, which is the measuring line, and is equal to 144 feet, 11 inches.

**ITINERARY MEASURE.**—400 Cubits —1 Stadium; 5 Stadia —1 Sabbath-day's Journey; 10 Stadia —1 Eastern Mile; 3 Eastern Miles —1 Parasang; 8 Parasangs —1 Day's Journey, or  $33\frac{1}{3}$  English Miles.

**DRY MEASURE.**—20 Grachal —1 Cab;  $1\frac{1}{2}$  Cabs —1 Gomor;  $3\frac{1}{3}$  Gomor —1 Seah; 3 Seahs —1 Ephah; 5 Ephahs —1 Leteeh; 2 Leteeh —1 Comer, or 2 Bushels, 1 pint, English.

**LIQUID MEASURE.**— $1\frac{1}{2}$  Caph —1 Log; 4 Logs —1 Cab; 3 Cabs —1 Hin; 2 Hins —1 Seah; 3 Seahs —1 Bath or

\* Although rial of old plate is not a coin, yet it is the denomination in which exchanges and invoices are reckoned.

† See *Kelly's Universal Cambist*.

Ephah; 10 Ephahs — 1 Chomer, Homer, or Corus, which equals 75 gallons, 5 pints, English Measure.

1 Talents — 113 lbs., 10 oz., 1 dwt., 10 grs; or 655714 grs.	1 Eas. Mile — 2432.2634 yards.
1 Maneh — 13114.28 grs.; or, 27.3214 oz.; or, 2.27678 lbs. T.	1 Stadium — 243.2263 "
1 Shekel — 218.57133 grs.	1 Sab. D. J. — 1216.1315 "
1 Schoenus — 145 ft. 11 in.; or, 1 " — 1751. inches.	1 Comer — 2 bu. 1 pt., Eng.; or 16.125 galls.; or 129. pta.
1 Pole — 175.1 "	1 Leteeh — 64.5 pinta.
1 Fathom — 87.55 ", or, 7 ft. 3 1/2 inches.	1 Ephah — 12.9 "
1 Cubit — 21.8875 inches.	1 Seah — 4.3 "
1 Span — 10.9437 "	1 Gomor — 1.29 "
1 Palm — 3.6477 "	1 Cab — .7166 "
1 Digit — .9119 "	1 Grachal — .0358 "
1 Day's Journey — 33 1/2 Eng-lish miles; or 58374.3216 yds.	1 Chomer — 15 galls., 5 pta., English; or 605 pinta.
1 Parasang — 7296.7902 "	1 Ephah — 60.5 pinta.
	1 Seah — 21.166 "
	1 Hin — 10.083 "
	1 Cab — 3.361 "
	1 Log — .8402 "
	1 Caph — .6301 "

1 Talent, silver — \$1589.61; of gold, — \$25415.27. 1 Maneh, silver — \$31.79; gold, — \$508.22. 1 Shekel, — \$0.529; gold, — \$8.47; all 24 carats fine, allowing no alloy.

*Time Table, for Banking and Equation, giving the number of days from any given date in one month, to the same date in any other month.*

A. D.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1849.												
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	272	303
Mar.	306	337	365	31	61	91	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May.	245	276	304	335	365	31	61	92	123	153	184	214
June.	214	245	273	304	334	365	30	61	92	122	153	183
July.	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

The number of days expiring between any two periods may be very easily ascertained in the foregoing table, by ascertaining the time between the first and second dates, and adding or subtracting the overplus, or deficit, minus 1. For example:

How long does a note run, dated January 4, and payable December 14?

In the table above, from Jan. 4, to Dec. 4, is 334 days; and 9 days more, added, excluding the latter date, make 343 days, the time that the note runs.

How long does a note run from December 4, to January 4?

In the left-hand column we find December, and opposite it, under the head Jan., we find 31 days, the time. The month of the first date must be sought in the column at the left.

For leap-year, one must be added to the number of days, when the month of February comes within the two dates.

TABLE  
Of Diameters and Areas of Circles.

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
1 in.	.7854	10 1/2	82.546	19 1/2	298.648	28 1/2	649.188	38	1134.11
1 1/8	1.2277	10 3/4	86.580	19 3/4	306.355	29	680.521	38 1/2	1149.08
1 1/4	1.7671	10 7/8	90.762	20	314.160	29 1/2	671.968	39	1164.15
1 1/2	2.4062	11	95.035	20 1/2	322.063	29 3/4	683.494	39 1/2	1179.33
2	3.1416	11 1/8	99.402	20 3/4	330.964	29 3/2	696.128	39 3/4	1194.59
2 1/8	3.9790	11 1/4	103.869	20 7/8	338.163	30	706.860	39 1/2	1209.95
2 1/4	4.9077	11 3/8	108.434	21	346.361	30 1/2	718.690	39 3/4	1225.43
2 1/2	5.9396	11 1/2	113.097	21 1/8	354.657	30 3/4	730.618	39 1/2	1240.98
3	7.0686	11 3/4	117.859	21 1/4	363.061	30 1/2	744.644	40	1256.64
3 1/8	8.2997	11 1/2	122.718	21 3/8	371.543	31	754.769	40 1/2	1272.39
3 1/4	9.6217	11 1/4	127.676	21 1/2	380.133	31 1/2	768.992	40 3/4	1288.26
3 1/2	11.044	11 1/8	132.732	21 3/4	388.832	31 3/4	779.312	40 1/2	1304.20
4	12.566	11 3/8	137.886	22	397.606	32	791.732	41	1320.26
4 1/8	14.186	11 1/4	143.139	22 1/8	406.493	32 1/2	804.249	41 1/2	1336.40
4 1/4	15.904	11 1/2	148.489	22 1/4	415.476	32 3/4	816.865	41 3/4	1352.65
4 1/2	17.720	11 3/4	153.938	22 3/8	424.567	33	829.578	41 1/2	1369.00
5	19.635	11 1/2	159.485	22 1/2	433.731	33 1/2	842.390	42	1385.44
5 1/8	21.647	11 1/4	165.130	22 3/4	443.004	33 3/4	855.20	42 1/2	1401.98
5 1/4	23.758	11 1/8	170.873	23	452.390	34	868.30	42 3/4	1418.62
5 1/2	25.967	11 3/8	176.715	23 1/8	461.864	34 1/2	881.41	42 1/2	1435.36
6	28.274	11 1/4	182.654	23 1/4	471.426	34 3/4	894.61	43	1452.20
6 1/8	30.679	11 1/2	188.692	23 3/8	481.106	35	907.92	43 1/2	1469.13
6 1/4	33.183	11 3/4	194.828	23 1/2	490.875	35 1/2	921.32	43 3/4	1486.17
6 1/2	35.784	11 1/2	201.062	23 3/4	500.741	35 3/4	934.82	43 1/2	1503.30
7	38.484	11 1/4	207.394	24	510.706	36	948.41	44	1520.53
7 1/8	41.282	11 1/8	213.825	24 1/8	520.769	36 1/2	962.11	44 1/2	1537.86
7 1/4	44.178	11 3/8	220.358	24 1/4	530.930	36 3/4	975.99	44 3/4	1555.28
7 1/2	47.173	11 1/2	226.980	24 3/8	541.189	36 1/2	989.90	44 1/2	1572.81
8	50.265	11 1/4	233.705	24 1/2	551.547	36 3/4	1003.70	45	1590.43
8 1/8	53.466	11 1/8	240.528	24 3/4	562.002	37	1017.57	45 1/2	1608.15
8 1/4	56.745	11 3/8	247.450	25	572.556	37 1/2	1032.06	45 3/4	1626.97
8 1/2	60.132	11 1/2	254.469	25 1/8	583.208	37 3/4	1046.30	45 1/2	1645.89
9	63.617	11 1/4	261.587	25 1/4	593.963	38	1060.73	46	1661.90
9 1/8	67.200	11 1/8	268.803	25 3/8	604.807	38 1/2	1075.21	46 1/2	1679.01
9 1/4	70.882	11 3/8	276.117	25 1/2	615.753	38 3/4	1089.79	46 3/4	1696.23
9 1/2	74.662	11 1/2	283.629	25 3/4	626.798	38 1/2	1104.46	46 1/2	1713.54
10	78.540	11 1/4	291.639	25 1/2	637.941	38 3/4	1119.24	47	1730.94

TABLE—Continued.

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
47 1/4	1753.45	57 1/4	2619.35	68 1/4	3658.44	85 1/4	5741.47	9 8	73.391
47 1/2	1772.05	58	2642.08	68 1/2	3685.29	86	5808.51	9 9	74.662
47 3/4	1790.76	58 1/4	2664.91	68 3/4	3712.24	86 1/2	5876.55	9 10	75.943
48	1809.56	58 1/2	2687.83	69	3739.28	87	5944.69	9 11	77.226
48 1/4	1828.46	58 3/4	2710.85	69 1/4	3766.43	87 1/2	6013.21	10	78.540
48 1/2	1847.45	59	2733.97	69 1/2	3793.67	88	6082.13	10 1	79.854
48 3/4	1866.55	59 1/4	2757.19	69 3/4	3821.02	88 1/2	6151.44	10 2	81.179
49	1885.74	59 1/2	2780.51	70	3848.46	89	6221.15	10 3	82.516
49 1/4	1905.03	59 3/4	2803.92	70 1/4	3875.99	89 1/2	6291.25	10 4	83.852
49 1/2	1924.42	60	2827.44	70 1/2	3903.63	90	6361.74	10 5	85.201
49 3/4	1943.91	60 1/4	2851.05	70 3/4	3931.36	90 1/2	6432.62	10 6	86.550
50	1963.50	60 1/2	2874.76	71	3959.20	91	6503.89	10 7	87.909
50 1/4	1983.18	60 3/4	2898.59	71 1/4	3987.13	91 1/2	6575.56	10 8	89.268
50 1/2	2002.96	61	2922.47	71 1/2	4015.16	92	6647.63	10 9	90.762
50 3/4	2022.84	61 1/4	2946.47	71 3/4	4043.28	92 1/2	6720.07	10 10	92.174
51	2042.82	61 1/2	2970.57	72	4071.51	93	6792.92	10 11	93.598
51 1/4	2062.90	61 3/4	2994.77	72 1/4	4129.25	93 1/2	6866.16	11	95.033
51 1/2	2083.07	62	3019.07	72 1/2	4185.39	94	6939.79	11 1	96.478
51 3/4	2103.35	62 1/4	3043.47	72 3/4	4242.92	94 1/2	7013.51	11 2	97.934
52	2123.73	62 1/2	3067.96	73	4300.85	95	7088.32	11 3	99.402
52 1/4	2144.19	62 3/4	3092.56	73 1/4	4359.15	95 1/2	7163.04	11 4	100.879
52 1/2	2164.75	63	3117.25	73 1/2	4417.87	ft. in.		11 5	102.368
52 3/4	2185.42	63 1/4	3142.04	73 3/4	4476.97	8		11 6	103.869
53	2206.18	63 1/2	3166.92	74	4536.47	8 1		11 7	105.379
53 1/4	2227.05	63 3/4	3191.91	74 1/4	4596.35	8 2		11 8	106.901
53 1/2	2248.01	64	3216.99	74 1/2	4656.63	8 3		11 9	108.424
53 3/4	2269.06	64 1/4	3242.17	74 3/4	4717.30	8 4		11 10	109.977
54	2290.23	64 1/2	3267.46	75	4778.27	8 5		11 11	111.531
54 1/4	2311.48	64 3/4	3292.83	75 1/4	4839.53	8 6		12	113.097
54 1/2	2332.83	65	3318.31	75 1/2	4901.68	8 7		12	114.732
54 3/4	2354.28	65 1/4	3343.98	75 3/4	4963.92	8 8		13	116.388
55	2375.83	65 1/2	3369.76	76	5026.56	8 9		13	118.065
55 1/4	2397.48	65 3/4	3395.63	76 1/4	5089.58	8 10		13	119.754
55 1/2	2419.23	66	3421.60	76 1/2	5153.00	8 11		14	121.464
55 3/4	2441.07	66 1/4	3447.16	76 3/4	5216.82	9		14	123.185
56	2463.01	66 1/2	3473.23	76 1/2	5281.02	9 1		15	124.927
56 1/4	2485.05	66 3/4	3499.39	76 3/4	5345.62	9 2		15	126.690
56 1/2	2507.19	67	3525.66	76 1/2	5410.62	9 3		16	128.464
56 3/4	2529.43	67 1/4	3552.01	76 3/4	5476.00	9 4		16	130.259
57	2551.76	67 1/2	3578.47	76 1/2	5541.78	9 5		17	132.074
57 1/4	2574.19	67 3/4	3605.03	76 3/4	5607.95	9 6		17	133.909
57 1/2	2596.72	68	3631.68	76 1/2	5674.51	9 7		18	135.764

Weight of a Lineal Foot of Square Rolled Iron, in lbs., from 1/4 to 12 inches square.

Size, in in.	Weight, in lbs.	Size, in in.	Weight, in lbs.	Size, in in.	Weight, in lbs.	Size, in in.	Weight, in lbs.	Size, in in.	Weight, in lbs.
1/4	.311	1	2.588	1 1/2	7.604	2 1/2	15.933	2 3/4	25.560
1/2	.475	1 1/4	3.380	1 3/4	8.926	2 3/4	17.112	2 7/8	27.989
3/4	.645	1 1/2	4.278	1 7/8	10.352	2 7/8	19.066	3	30.416
1	1.230	1 3/4	5.380	2	11.883	3	21.120	3 1/8	33.010
1 1/4	1.901	2	6.390	2 1/4	13.520	3 1/8	23.292	3 1/4	36.704

# WEIGHT OF DIFFERENT BODIES OF IRON. 281

TABLE—Continued.

Size, in in.	Wei't, in lbs.	Size, in in.	Wei't, in lbs.	Size, in in.	Wei't, in lbs.	Size, in in.	Wei't, in lbs.	Size, in in.	Wei't, in lbs.
3 $\frac{3}{8}$	86.503	4 $\frac{1}{8}$	72.205	5 $\frac{1}{4}$	111.756	7 $\frac{1}{2}$	203.024	10	337.920
3 $\frac{1}{2}$	41.408	4 $\frac{1}{4}$	76.364	5 $\frac{1}{2}$	116.671	8	216.336	10 $\frac{1}{2}$	355.126
3 $\frac{7}{8}$	44.418	4 $\frac{3}{8}$	80.333	6	121.664	8 $\frac{1}{2}$	230.068	10 $\frac{3}{4}$	372.672
3 $\frac{15}{16}$	47.534	5	84.430	6 $\frac{1}{4}$	132.040	9	244.230	10 $\frac{1}{2}$	390.628
3 $\frac{1}{2}$	50.756	5 $\frac{1}{4}$	88.784	6 $\frac{1}{2}$	142.816	9 $\frac{1}{2}$	258.900	11	408.960
4	54.084	5 $\frac{1}{2}$	93.168	6 $\frac{3}{4}$	154.012	10	273.792	11 $\frac{1}{2}$	427.812
4 $\frac{1}{8}$	57.517	5 $\frac{3}{4}$	97.657	7	165.633	10 $\frac{1}{2}$	289.220	11 $\frac{3}{4}$	447.024
4 $\frac{1}{4}$	61.055	6	102.240	7 $\frac{1}{4}$	177.672	11	305.056	11 $\frac{1}{2}$	466.624
4 $\frac{3}{8}$	64.700	6 $\frac{1}{4}$	106.953	7 $\frac{1}{2}$	190.136	11 $\frac{3}{4}$	321.332	12	486.656
4 $\frac{1}{2}$	68.448	6 $\frac{3}{8}$							

Weight of Round Rolled Iron, 1 foot long, and from  $\frac{1}{4}$  to 12 inches in diameter.

Dia., in in.	Wei't, in lbs.	Dia., in in.	Wei't, in lbs.	Dia., in in.	Wei't, in lbs.	Dia., in in.	Wei't, in lbs.	Dia., in in.	Wei't, in lbs.
$\frac{1}{4}$	.165	2 $\frac{1}{4}$	11.988	3 $\frac{3}{8}$	39.864	5 $\frac{1}{2}$	84.001	8 $\frac{1}{2}$	203.269
$\frac{3}{8}$	.373	2 $\frac{1}{2}$	13.440	4	43.464	5 $\frac{3}{4}$	87.776	9	215.040
$\frac{1}{2}$	.668	2 $\frac{3}{4}$	14.975	4 $\frac{1}{4}$	45.174	6	91.684	9 $\frac{1}{2}$	227.162
$\frac{5}{8}$	1.043	2 $\frac{7}{8}$	16.688	4 $\frac{3}{8}$	47.952	6 $\frac{1}{4}$	95.552	9 $\frac{3}{4}$	239.600
$\frac{3}{4}$	1.493	3	18.293	4 $\frac{1}{2}$	50.815	6 $\frac{3}{8}$	103.704	9 $\frac{1}{2}$	252.376
$\frac{7}{8}$	2.032	3 $\frac{1}{8}$	20.076	4 $\frac{3}{4}$	53.760	6 $\frac{1}{2}$	112.160	10	266.288
1	2.654	3 $\frac{1}{4}$	21.944	5	56.788	6 $\frac{3}{4}$	120.960	10 $\frac{1}{2}$	278.924
1 $\frac{1}{8}$	3.260	3 $\frac{3}{8}$	23.888	5 $\frac{1}{4}$	59.900	7	130.048	10 $\frac{3}{4}$	292.688
1 $\frac{1}{4}$	4.172	3 $\frac{1}{2}$	25.926	5 $\frac{3}{8}$	63.094	7 $\frac{1}{4}$	139.544	10 $\frac{1}{2}$	306.800
1 $\frac{1}{2}$	5.019	3 $\frac{3}{4}$	28.040	5 $\frac{1}{2}$	66.752	7 $\frac{1}{2}$	149.328	11	321.216
1 $\frac{3}{8}$	5.972	3 $\frac{7}{8}$	30.240	5 $\frac{3}{4}$	69.731	7 $\frac{3}{8}$	159.456	11 $\frac{1}{2}$	336.004
1 $\frac{1}{2}$	7.010	3 $\frac{1}{2}$	32.512	5 $\frac{1}{2}$	73.172	8	169.856	11 $\frac{3}{4}$	351.104
1 $\frac{3}{4}$	8.128	3 $\frac{3}{4}$	34.886	5 $\frac{3}{4}$	76.700	8 $\frac{1}{4}$	180.696	11 $\frac{1}{2}$	366.536
1 $\frac{7}{8}$	9.333	3 $\frac{7}{8}$	37.332	5 $\frac{7}{8}$	80.304	8 $\frac{3}{8}$	191.568	12	382.208
2	10.616								

Weight, in lbs., of different bodies of Cast Iron, 1 foot in length, and from  $\frac{1}{2}$  to 12 inches diameter or side.

Side, or di. inch.	Equa.	Hexa- gon.	Octa- gon.	Circle.	Side, or di. inch.	Equa.	Hexa- gon.	Octa- gon.	Circle.
$\frac{1}{2}$	.781	.675	.650	.612	3 $\frac{1}{4}$	33.009	28.565	27.475	25.921
$\frac{3}{4}$	1.756	1.528	1.471	1.387	3 $\frac{1}{2}$	38.281	33.131	31.818	30.065
1	3.126	2.703	2.603	2.454	3 $\frac{3}{4}$	43.943	38.031	36.581	34.512
1 $\frac{1}{4}$	4.881	4.225	4.065	3.854	4	50.000	43.271	41.621	39.263
1 $\frac{1}{2}$	7.031	6.085	5.856	5.521	4 $\frac{1}{4}$	56.443	48.353	46.990	44.331
1 $\frac{3}{4}$	9.568	8.281	7.971	7.515	4 $\frac{1}{2}$	63.281	54.769	52.681	49.700
2	12.520	10.815	10.412	9.815	4 $\frac{3}{4}$	70.506	61.021	58.696	55.375
2 $\frac{1}{4}$	15.818	13.990	13.168	12.425	5	78.125	67.515	65.040	61.259
2 $\frac{1}{2}$	19.581	16.900	16.256	15.337	5 $\frac{1}{4}$	86.131	74.549	71.701	67.709
2 $\frac{3}{4}$	23.631	20.450	19.671	18.569	5 $\frac{1}{2}$	94.531	81.815	78.696	74.243
3	28.125	24.340	23.412	22.087	5 $\frac{3}{4}$	103.318	89.421	86.015	81.126

TABLE—Continued.

Side, or di.	Squa.	Hexa- gon.	Octa- gon.	Circle.	Side, or di.	Squa.	Hexa- gon.	Octa- gon.	Circle.
inch.					inch.				
5	112.500	97.248	93.696	88.354	9½	266.781	231.418	222.600	210.800
5½	122.058	105.640	101.621	96.871	9¾	282.031	244.100	234.798	221.506
6	132.031	114.271	109.948	103.696	9⅝	296.968	257.106	247.315	233.318
6½	142.881	123.231	118.534	111.825	10	312.500	270.471	260.168	245.437
7	153.125	132.528	127.478	120.372	10½	328.318	284.169	273.341	257.869
7½	161.256	142.162	136.743	128.998	10¾	344.531	298.193	286.828	270.593
8	175.781	152.037	146.337	138.066	10⅝	351.131	312.559	300.645	283.633
8½	187.693	162.449	156.269	147.415	11	378.125	327.268	314.796	296.978
9	200.000	173.099	166.503	157.078	11½	393.216	342.315	329.268	310.621
9½	212.693	184.087	177.071	167.049	11¾	410.281	357.693	344.062	324.587
10	225.781	195.412	187.365	177.328	11⅝	429.028	373.326	359.187	338.866
10½	239.256	207.078	199.127	187.912	12	450.000	389.475	374.613	353.428
11	253.125	219.078	210.721	199.203					

Weight of a lineal foot of Flat Bar Iron, in lbs., from ½ to 5½ inches in width, and from ½ to 5 inches in thickness.

Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.	Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.	Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.	Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.
½	½	0.211	½	½	2.375	1	1	4.435	1½	1½	9.610
½	½	0.422	1	1	2.850	1	1	5.069	1½	1½	0.792
½	½	0.634	1	1	3.326	1	1	5.703	1½	1½	1.584
½	½	0.846	1	1	3.802	1	1	6.337	1½	1½	2.376
½	½	0.528	1½	1½	0.528	1	1	6.970	1½	1½	3.168
½	½	0.792	1½	1½	1.056	1	1	0.686	1½	1½	3.960
½	½	1.056	1½	1½	1.584	1	1	1.372	1½	1½	4.752
½	½	0.316	1½	1½	2.112	1	1	2.059	1½	1½	5.544
½	½	0.633	1½	1½	2.640	1	1	2.746	1	1	6.336
½	½	0.950	1½	1½	3.168	1	1	3.432	1½	1½	7.128
½	½	1.265	1½	1½	3.696	1	1	4.119	1½	1½	7.921
½	½	1.584	1	1	4.224	1	1	4.805	1½	1½	8.713
½	½	0.369	1½	1½	4.752	1	1	5.492	1½	1½	9.505
½	½	0.738	1½	1½	0.580	1½	1½	6.178	1½	1½	10.297
½	½	1.103	1½	1½	1.161	1½	1½	6.864	1½	1½	11.089
½	½	1.477	1½	1½	1.742	1½	1½	7.551	1½	1½	0.845
½	½	1.846	1½	1½	2.325	1½	1½	8.237	1½	1½	1.639
½	½	2.217	1½	1½	2.904	1½	1½	0.739	1½	1½	2.534
½	½	0.422	1½	1½	3.484	1½	1½	1.479	1½	1½	3.379
½	½	0.845	1½	1½	4.065	1½	1½	2.218	1½	1½	4.224
½	½	1.267	1	1	4.646	1½	1½	2.957	1½	1½	5.069
½	½	1.690	1½	1½	5.227	1½	1½	3.696	1½	1½	5.914
½	½	2.112	1½	1½	5.808	1½	1½	4.435	1	1	6.758
½	½	2.534	1½	1½	0.633	1½	1½	5.178	1½	1½	7.604
½	½	2.956	1½	1½	1.265	1	1	5.914	1½	1½	8.448
½	½	0.475	1½	1½	1.900	1½	1½	6.653	1½	1½	9.294
½	½	0.960	1½	1½	2.535	1½	1½	7.392	1½	1½	10.138
½	½	1.425	1½	1½	3.168	1½	1½	8.132	1½	1½	10.983
½	½	1.901	1½	1½	3.802	1½	1½	8.871	1½	1½	11.828
									2½	2½	12.673
									2½	2½	0.898
									2½	2½	1.795
									2½	2½	2.693
									2½	2½	3.591
									2½	2½	4.488
									2½	2½	5.386
									2½	2½	6.283
									2½	2½	7.181
									2½	2½	8.079
									2½	2½	8.977
									2½	2½	9.874
									2½	2½	10.772
									2½	2½	11.670
									2½	2½	12.567
									2½	2½	13.465
									2½	2½	14.362
									2½	2½	0.360
									2½	2½	1.900
									2½	2½	2.951
									2½	2½	3.902
									2½	2½	4.752
									2½	2½	5.703
									2½	2½	6.653
									2½	2½	7.604
									2½	2½	8.554
									2½	2½	9.505
									2½	2½	10.455
									2½	2½	11.406



*Weight of a lineal foot of Cast Iron Pipes, or Cylinders, in lbs., the thickness and bore being given.*

Bore, in in.	Thick. in in.	Weight, in lbs.	Bore, in in.	Thick. in in.	Weight, in lbs.	Bore, in in.	Thick. in in.	Weight, in lbs.	Bore, in in.	Thick. in in.	Weight, in lbs.
1	$\frac{1}{8}$	3.06	7	$\frac{1}{8}$	43.68	12 $\frac{1}{2}$	$\frac{1}{8}$	77.36	17 $\frac{1}{2}$	$\frac{1}{8}$	88.28
$1\frac{1}{8}$	$\frac{1}{8}$	5.05		$\frac{1}{4}$	53.30		$\frac{1}{4}$	93.70	18	$\frac{1}{8}$	111.06
$1\frac{1}{4}$	$\frac{1}{8}$	3.67		$\frac{3}{8}$	63.18		$\frac{3}{8}$	110.48		$\frac{1}{4}$	134.16
$1\frac{1}{2}$	$\frac{1}{8}$	6.		$\frac{1}{2}$	86.08		$\frac{1}{2}$	127.42		$\frac{3}{8}$	157.59
$1\frac{3}{4}$	$\frac{1}{8}$	6.89	7 $\frac{1}{2}$	$\frac{5}{8}$	46.80	13	$\frac{1}{8}$	63.70	19	$\frac{1}{8}$	181.33
$1\frac{7}{8}$	$\frac{1}{8}$	9.80		$\frac{3}{4}$	56.96		$\frac{3}{8}$	80.40		$\frac{1}{4}$	114.10
$2$	$\frac{1}{8}$	7.80		$\frac{7}{8}$	67.60		$\frac{1}{2}$	97.40		$\frac{3}{8}$	127.84
$2\frac{1}{8}$	$\frac{1}{8}$	11.04		$1$	78.39		$\frac{3}{4}$	114.73		$\frac{1}{2}$	161.90
$2\frac{1}{4}$	$\frac{1}{8}$	8.74	8	$1\frac{1}{8}$	39.23	13 $\frac{1}{2}$	$\frac{1}{8}$	132.26	20	$\frac{1}{8}$	183.94
$2\frac{1}{2}$	$\frac{1}{8}$	12.23		$1\frac{1}{4}$	49.93		$\frac{1}{4}$	66.14		$\frac{1}{4}$	120.94
$2\frac{3}{4}$	$\frac{1}{8}$	9.65		$1\frac{3}{8}$	60.48		$\frac{3}{8}$	83.46		$\frac{3}{8}$	145.30
$2\frac{7}{8}$	$\frac{1}{8}$	12.48		$1\frac{1}{2}$	71.76		$\frac{1}{2}$	101.08		$\frac{1}{2}$	170.47
$3$	$\frac{1}{8}$	10.87	8 $\frac{1}{2}$	$1\frac{5}{8}$	83.38	14	$\frac{1}{8}$	118.97	21	$\frac{1}{8}$	196.92
$3\frac{1}{8}$	$\frac{1}{8}$	14.66		$1\frac{3}{4}$	41.64		$\frac{3}{8}$	127.28		$\frac{1}{4}$	126.53
$3\frac{1}{4}$	$\frac{1}{8}$	19.05		$1\frac{7}{8}$	53.68		$\frac{1}{2}$	68.64		$\frac{3}{8}$	152.53
$3\frac{3}{4}$	$\frac{1}{8}$	11.54		$2$	64.37		$\frac{3}{4}$	86.55		$\frac{1}{2}$	179.02
$3\frac{1}{2}$	$\frac{1}{8}$	15.91	9	$2\frac{1}{8}$	76.12	14 $\frac{1}{2}$	$\frac{1}{8}$	104.76	22	$\frac{1}{8}$	206.90
$3\frac{3}{4}$	$\frac{1}{8}$	20.59		$2\frac{1}{4}$	88.20		$\frac{1}{4}$	128.20		$\frac{1}{4}$	132.60
$4$	$\frac{1}{8}$	12.28		$2\frac{3}{8}$	44.11		$\frac{3}{8}$	142.16		$\frac{3}{8}$	159.84
$4\frac{1}{8}$	$\frac{1}{8}$	17.15		$2\frac{1}{2}$	56.16	15	$\frac{1}{2}$	71.07	23	$\frac{1}{8}$	187.80
$4\frac{1}{4}$	$\frac{1}{8}$	22.15	9 $\frac{1}{2}$	$2\frac{5}{8}$	68.		$\frac{3}{4}$	89.61		$\frac{1}{4}$	215.62
$4\frac{1}{2}$	$\frac{1}{8}$	27.56		$2\frac{7}{8}$	80.50		$1$	108.46		$\frac{3}{8}$	139.60
$4\frac{3}{4}$	$\frac{1}{8}$	18.40		$3$	96.38		$1\frac{1}{8}$	127.60		$\frac{1}{2}$	167.24
$4\frac{7}{8}$	$\frac{1}{8}$	23.73	10	$3\frac{1}{8}$	46.50	15 $\frac{1}{2}$	$\frac{1}{8}$	147.03	24	$\frac{1}{8}$	196.46
$5$	$\frac{1}{8}$	29.64		$3\frac{1}{4}$	58.93		$\frac{1}{4}$	73.73		$\frac{1}{4}$	226.28
$5\frac{1}{8}$	$\frac{1}{8}$	19.66		$3\frac{3}{8}$	71.71		$\frac{3}{8}$	92.66		$\frac{3}{8}$	144.77
$5\frac{1}{4}$	$\frac{1}{8}$	25.37		$3\frac{1}{2}$	84.70	16	$\frac{1}{2}$	119.10	25	$\frac{1}{8}$	174.62
$5\frac{1}{2}$	$\frac{1}{8}$	31.20	10 $\frac{1}{2}$	$3\frac{5}{8}$	97.98		$\frac{3}{4}$	131.96		$\frac{1}{4}$	204.78
$5\frac{3}{4}$	$\frac{1}{8}$	20.90		$3\frac{7}{8}$	48.98		$1$	151.92		$\frac{3}{8}$	236.28
$5\frac{7}{8}$	$\frac{1}{8}$	26.83		$4$	62.02	16 $\frac{1}{2}$	$\frac{1}{8}$	75.96	26	$\frac{1}{8}$	150.55
$6$	$\frac{1}{8}$	33.07	11	$4\frac{1}{8}$	75.33		$\frac{1}{4}$	96.73		$\frac{1}{4}$	181.92
$6\frac{1}{8}$	$\frac{1}{8}$	22.05		$4\frac{1}{4}$	88.98		$\frac{3}{8}$	115.78		$\frac{3}{8}$	213.28
$6\frac{1}{4}$	$\frac{1}{8}$	28.28		$4\frac{3}{8}$	102.90		$\frac{1}{2}$	136.15		$\frac{1}{2}$	245.08
$6\frac{1}{2}$	$\frac{1}{8}$	34.94		$4\frac{1}{2}$	51.46	17	$\frac{1}{8}$	156.82	27	$\frac{1}{8}$	156.97
$6\frac{3}{4}$	$\frac{1}{8}$	23.35	11 $\frac{1}{2}$	$4\frac{5}{8}$	66.08		$\frac{1}{4}$	78.40		$\frac{1}{4}$	180.23
$6\frac{7}{8}$	$\frac{1}{8}$	29.85		$4\frac{3}{4}$	78.99		$\frac{3}{8}$	98.78		$\frac{3}{8}$	221.94
$7$	$\frac{1}{8}$	36.78		$4\frac{7}{8}$	96.24		$\frac{1}{2}$	119.48		$\frac{1}{2}$	264.36
$7\frac{1}{8}$	$\frac{1}{8}$	24.49	12	$5$	108.84	17 $\frac{1}{2}$	$\frac{1}{8}$	140.40	28	$\frac{1}{8}$	186.63
$7\frac{1}{4}$	$\frac{1}{8}$	31.40		$5\frac{1}{8}$	53.88		$\frac{1}{4}$	161.82		$\frac{1}{4}$	230.56
$7\frac{1}{2}$	$\frac{1}{8}$	38.58		$5\frac{1}{4}$	68.14		$\frac{3}{8}$	80.87		$\frac{3}{8}$	264.66
$7\frac{3}{4}$	$\frac{1}{8}$	25.70		$5\frac{3}{8}$	82.68		$\frac{1}{2}$	101.82		$\frac{1}{2}$	304.04
$7\frac{7}{8}$	$\frac{1}{8}$	32.91	12 $\frac{1}{2}$	$5\frac{1}{2}$	97.44	18	$\frac{3}{4}$	123.14	29	$\frac{1}{8}$	239.08
$8$	$\frac{1}{8}$	40.43		$5\frac{5}{8}$	112.68		$1$	144.78		$\frac{1}{4}$	274.56
$8\frac{1}{8}$	$\frac{1}{8}$	26.94		$5\frac{3}{4}$	66.84		$\frac{1}{8}$	166.60		$\frac{3}{8}$	211.32
$8\frac{1}{4}$	$\frac{1}{8}$	34.34		$5\frac{7}{8}$	71.19	19	$\frac{1}{4}$	83.20		$\frac{1}{2}$	247.62
$8\frac{1}{2}$	$\frac{1}{8}$	42.28	13	$6$	86.40		$\frac{3}{8}$	104.83		$\frac{1}{2}$	284.28
$8\frac{3}{4}$	$\frac{1}{8}$	29.40		$6\frac{1}{8}$	101.83		$\frac{1}{2}$	126.79		$\frac{3}{8}$	218.70
$8\frac{7}{8}$	$\frac{1}{8}$	37.44		$6\frac{1}{4}$	117.60		$\frac{3}{4}$	149.02		$\frac{1}{2}$	258.30
$9$	$\frac{1}{8}$	45.94	13 $\frac{1}{2}$	$6\frac{3}{8}$	58.82	20	$\frac{1}{8}$	171.60	30	$\frac{1}{8}$	294.02
$9\frac{1}{8}$	$\frac{1}{8}$	31.22		$6\frac{1}{2}$	74.28		$\frac{1}{4}$	85.73		$\frac{1}{4}$	226.20
$9\frac{1}{4}$	$\frac{1}{8}$	40.56		$6\frac{5}{8}$	80.06		$\frac{3}{8}$	107.98		$\frac{3}{8}$	264.79
$9\frac{1}{2}$	$\frac{1}{8}$	49.60		$6\frac{3}{4}$	106.14		$\frac{1}{2}$	130.48		$\frac{1}{2}$	308.86
$9\frac{3}{4}$	$\frac{1}{8}$	58.98	14	$6\frac{7}{8}$	123.62	21	$\frac{3}{4}$	158.30		$\frac{1}{2}$	343.20
$10$	$\frac{1}{8}$	34.82		$7$	61.26		$1$	176.58		$\frac{3}{8}$	



$\frac{1}{2}$  Eagle [1795].—\$5.25.



Eagle.—\$10.50.



$\frac{1}{2}$  Eagle.—\$5.00.



5 Sovereigns.—\$24.25.



$\frac{1}{2}$  Sovereigns.—2.42.

$\frac{1}{4}$  Eagle—\$2.50. Sovereign.—\$4.85.



Guinea.—\$5.  
19



40 Francs.—\$7.66



Louis d'or.—\$4.50

286 GOLD COIN.—*American, English and French.*



40 Francs.—\$7.60. Double Sovereign.—\$9.60. Eagle.—\$10.00.



Bechtler pieces.—\$2.37.

20 Francs.—\$3.83. Sover'n.—\$2.42.



Guinea.—\$5.00.



\$4.75.



Louis d'or.—4.32.



American Gold Dollar.



16th Doubloons.—\$1.



\$1.20.



Sovereign.—\$4.85.



Double Louis d'or.—\$8.50.



Sovereign.—\$4.85.



Pistole.—\$3.90



92 cents. 1-16 Doub.—93c.  $\frac{1}{8}$  Doub.—\$1.90.



$\frac{1}{4}$  Guineas.—\$2.50



20 Francs.—\$3.82.



Sovereign.—\$4.85.



6 Francs.—\$1.12.



$\frac{1}{4}$  Guinea.—\$1.66



Sovereign.—\$4.85.



$\frac{1}{2}$  Doubloon.—\$3.75.



$\frac{1}{4}$  Doubleton.—\$3.80.



$\frac{1}{2}$  Doubloon.—\$7.75.



$\frac{1}{2}$  Doubloon.—\$8.



$\frac{1}{4}$  Doub.—\$3.80.



Doubloon.—\$15.55-60.



Doubloon.—\$15.50.



Doubloon.—\$15.55



Dobraon.—\$34.



Doubloon.—\$15.55.



Doubloon.—\$15.55.



20 Francs.—\$3.83.



4 Doublon.—\$4.



Sovereign.—\$4.85



$\frac{1}{4}$  Doub.—\$3.80.



$\frac{1}{4}$  Doubloon.—\$7.75.



Double Fred.d'or.—\$7.80.



Doubloon.—\$15.55.



Doubloon.—\$15.55.



Doubloon.—\$15.55.



Doubloon.—\$15.55-60.



Moidore [Brazil] \$4.80.



$\frac{1}{4}$  Joe.—\$8.50.



\$1.96

TABLE—Continued.

Side, or di.	Squa.	Hexa- gon.	Octa- gon.	Circlo.	Side, or di.	Squa.	Hexa- gon.	Octa- gon.	Circlo.
inch.					inch.				
6	112.500	97.268	93.066	88.354	9	266.781	231.418	222.600	210.800
6	132.058	105.640	101.621	96.871	9	282.031	244.100	234.798	221.506
6	132.031	114.271	109.948	103.696	9	286.968	257.106	247.315	233.318
6	142.881	123.231	118.634	111.825	10	312.500	270.471	260.168	245.437
7	153.125	132.528	127.478	120.373	10	328.318	284.169	273.341	257.869
7	161.266	142.163	136.743	128.936	10	344.531	298.193	286.826	270.593
7	175.781	152.037	146.237	138.056	10	351.131	312.560	300.645	283.633
7	187.693	162.449	156.269	147.415	11	378.125	327.268	314.796	296.978
8	200.000	173.099	166.603	157.078	11	393.216	342.315	329.268	310.621
8	212.688	184.087	177.071	167.049	11	410.231	357.693	344.062	324.587
8	225.781	195.412	187.365	177.328	11	428.023	373.326	359.187	338.866
8	239.256	207.078	199.127	187.912	12	450.000	389.475	374.613	353.426
9	253.125	219.078	210.721	199.203					

Weight of a lineal foot of Flat Bar Iron, in lbs., from  $\frac{1}{8}$  to  $5\frac{1}{2}$  inches in width, and from  $\frac{1}{8}$  to 5 inches in thickness.

Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.	Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.	Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.	Wt., in in.	T <sup>h</sup> , in in.	Wt., in lbs.
$\frac{1}{8}$	$\frac{1}{8}$	0.211	$\frac{1}{8}$	$\frac{1}{8}$	2.375	$\frac{1}{8}$	$\frac{1}{8}$	4.435	$\frac{1}{8}$	$\frac{1}{8}$	12.673
$\frac{1}{8}$	$\frac{1}{8}$	0.422	$\frac{1}{8}$	$\frac{1}{8}$	2.850	$\frac{1}{8}$	$\frac{1}{8}$	5.069	$\frac{1}{8}$	$\frac{1}{8}$	0.898
$\frac{1}{8}$	$\frac{1}{8}$	0.634	$\frac{1}{8}$	$\frac{1}{8}$	3.326	$\frac{1}{8}$	$\frac{1}{8}$	5.703	$\frac{1}{8}$	$\frac{1}{8}$	1.796
$\frac{1}{8}$	$\frac{1}{8}$	0.264	$\frac{1}{8}$	$\frac{1}{8}$	3.802	$\frac{1}{8}$	$\frac{1}{8}$	6.337	$\frac{1}{8}$	$\frac{1}{8}$	2.693
$\frac{1}{8}$	$\frac{1}{8}$	0.528	$\frac{1}{8}$	$\frac{1}{8}$	0.528	$\frac{1}{8}$	$\frac{1}{8}$	6.970	$\frac{1}{8}$	$\frac{1}{8}$	3.591
$\frac{1}{8}$	$\frac{1}{8}$	0.792	$\frac{1}{8}$	$\frac{1}{8}$	1.056	$\frac{1}{8}$	$\frac{1}{8}$	0.686	$\frac{1}{8}$	$\frac{1}{8}$	4.488
$\frac{1}{8}$	$\frac{1}{8}$	1.056	$\frac{1}{8}$	$\frac{1}{8}$	1.584	$\frac{1}{8}$	$\frac{1}{8}$	1.372	$\frac{1}{8}$	$\frac{1}{8}$	5.386
$\frac{1}{8}$	$\frac{1}{8}$	0.316	$\frac{1}{8}$	$\frac{1}{8}$	2.112	$\frac{1}{8}$	$\frac{1}{8}$	2.059	$\frac{1}{8}$	$\frac{1}{8}$	6.283
$\frac{1}{8}$	$\frac{1}{8}$	0.633	$\frac{1}{8}$	$\frac{1}{8}$	2.640	$\frac{1}{8}$	$\frac{1}{8}$	2.746	$\frac{1}{8}$	$\frac{1}{8}$	7.181
$\frac{1}{8}$	$\frac{1}{8}$	0.950	$\frac{1}{8}$	$\frac{1}{8}$	3.168	$\frac{1}{8}$	$\frac{1}{8}$	3.432	$\frac{1}{8}$	$\frac{1}{8}$	8.079
$\frac{1}{8}$	$\frac{1}{8}$	1.265	$\frac{1}{8}$	$\frac{1}{8}$	3.696	$\frac{1}{8}$	$\frac{1}{8}$	4.119	$\frac{1}{8}$	$\frac{1}{8}$	8.977
$\frac{1}{8}$	$\frac{1}{8}$	1.584	$\frac{1}{8}$	$\frac{1}{8}$	4.224	$\frac{1}{8}$	$\frac{1}{8}$	4.805	$\frac{1}{8}$	$\frac{1}{8}$	9.874
$\frac{1}{8}$	$\frac{1}{8}$	0.339	$\frac{1}{8}$	$\frac{1}{8}$	4.752	$\frac{1}{8}$	$\frac{1}{8}$	5.492	$\frac{1}{8}$	$\frac{1}{8}$	10.772
$\frac{1}{8}$	$\frac{1}{8}$	0.738	$\frac{1}{8}$	$\frac{1}{8}$	0.580	$\frac{1}{8}$	$\frac{1}{8}$	6.178	$\frac{1}{8}$	$\frac{1}{8}$	11.670
$\frac{1}{8}$	$\frac{1}{8}$	1.103	$\frac{1}{8}$	$\frac{1}{8}$	1.161	$\frac{1}{8}$	$\frac{1}{8}$	6.864	$\frac{1}{8}$	$\frac{1}{8}$	12.567
$\frac{1}{8}$	$\frac{1}{8}$	1.477	$\frac{1}{8}$	$\frac{1}{8}$	1.742	$\frac{1}{8}$	$\frac{1}{8}$	7.551	$\frac{1}{8}$	$\frac{1}{8}$	13.465
$\frac{1}{8}$	$\frac{1}{8}$	1.846	$\frac{1}{8}$	$\frac{1}{8}$	2.325	$\frac{1}{8}$	$\frac{1}{8}$	8.237	$\frac{1}{8}$	$\frac{1}{8}$	14.362
$\frac{1}{8}$	$\frac{1}{8}$	2.217	$\frac{1}{8}$	$\frac{1}{8}$	2.904	$\frac{1}{8}$	$\frac{1}{8}$	0.739	$\frac{1}{8}$	$\frac{1}{8}$	0.950
$\frac{1}{8}$	$\frac{1}{8}$	0.422	$\frac{1}{8}$	$\frac{1}{8}$	3.484	$\frac{1}{8}$	$\frac{1}{8}$	1.479	$\frac{1}{8}$	$\frac{1}{8}$	1.900
$\frac{1}{8}$	$\frac{1}{8}$	0.845	$\frac{1}{8}$	$\frac{1}{8}$	4.065	$\frac{1}{8}$	$\frac{1}{8}$	2.218	$\frac{1}{8}$	$\frac{1}{8}$	2.851
$\frac{1}{8}$	$\frac{1}{8}$	1.267	$\frac{1}{8}$	$\frac{1}{8}$	4.646	$\frac{1}{8}$	$\frac{1}{8}$	2.957	$\frac{1}{8}$	$\frac{1}{8}$	3.802
$\frac{1}{8}$	$\frac{1}{8}$	1.690	$\frac{1}{8}$	$\frac{1}{8}$	5.227	$\frac{1}{8}$	$\frac{1}{8}$	3.696	$\frac{1}{8}$	$\frac{1}{8}$	4.752
$\frac{1}{8}$	$\frac{1}{8}$	2.112	$\frac{1}{8}$	$\frac{1}{8}$	5.808	$\frac{1}{8}$	$\frac{1}{8}$	4.435	$\frac{1}{8}$	$\frac{1}{8}$	5.703
$\frac{1}{8}$	$\frac{1}{8}$	2.534	$\frac{1}{8}$	$\frac{1}{8}$	0.633	$\frac{1}{8}$	$\frac{1}{8}$	5.178	$\frac{1}{8}$	$\frac{1}{8}$	6.653
$\frac{1}{8}$	$\frac{1}{8}$	2.956	$\frac{1}{8}$	$\frac{1}{8}$	1.266	$\frac{1}{8}$	$\frac{1}{8}$	5.914	$\frac{1}{8}$	$\frac{1}{8}$	7.604
$\frac{1}{8}$	$\frac{1}{8}$	0.475	$\frac{1}{8}$	$\frac{1}{8}$	1.900	$\frac{1}{8}$	$\frac{1}{8}$	6.653	$\frac{1}{8}$	$\frac{1}{8}$	8.554
$\frac{1}{8}$	$\frac{1}{8}$	0.950	$\frac{1}{8}$	$\frac{1}{8}$	2.535	$\frac{1}{8}$	$\frac{1}{8}$	7.393	$\frac{1}{8}$	$\frac{1}{8}$	9.505
$\frac{1}{8}$	$\frac{1}{8}$	1.425	$\frac{1}{8}$	$\frac{1}{8}$	3.168	$\frac{1}{8}$	$\frac{1}{8}$	8.132	$\frac{1}{8}$	$\frac{1}{8}$	10.455
$\frac{1}{8}$	$\frac{1}{8}$	1.901	$\frac{1}{8}$	$\frac{1}{8}$	3.802	$\frac{1}{8}$	$\frac{1}{8}$	8.871	$\frac{1}{8}$	$\frac{1}{8}$	11.406

### WEIGHTS OF FLAT BAR IRON.

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**TABLE—Continued**

	Wi., in in.	Trk's, in in.	Wt., in lbs.		Wi., in in.	Trk's, in in.	Wt., in lbs.		Wi., in in.	Trk's, in in.	Wt., in lbs.		Wi., in in.	Trk's, in in.	Wt., in lbs.
2%	1%	12.356	1%	12.199	2%	26.719	1%	19.221	1%	17.953			1%	17.953	
	1%	13.307	1%	13.908	3%	1.287	1%	20.699	1%	21.544			1%	21.544	
	1%	14.257	1%	14.417	3%	2.638	1%	22.178	1%	23.135			1%	23.135	
	2%	15.208	1%	15.526	3%	3.893	2%	23.656	2%	28.775			2%	28.775	
	2%	16.158	1%	16.635	3%	5.089	2%	25.133	2%	32.316			2%	32.316	
2%	3%	1.003	2%	17.744	3%	6.337	2%	29.570	2%	35.907			2%	35.907	
	3%	2.006	2%	18.863	3%	7.604	2%	32.587	2%	39.497			2%	39.497	
	3%	3.009	2%	19.962	3%	8.871	3%	35.485	2%	43.088			2%	43.088	
	3%	4.013	2%	21.071	1%	10.138	3%	38.441	3%	46.679			3%	46.679	
	3%	5.016	2%	22.180	1%	11.406	3%	41.394	3%	50.269			3%	50.269	
2%	3%	6.019	3%	1.162	1%	12.678	3%	44.347	3%	53.860			3%	53.860	
	3%	7.022	3%	2.323	1%	13.940	3%	47.299	4%	57.450			4%	57.450	
	3%	8.025	3%	3.485	1%	15.202	3%	50.252	3%	61.041			3%	61.041	
	1%	9.028	3%	4.647	1%	16.475	3%	53.204	3%	71.632			3%	71.632	
	1%	10.032	3%	5.808	1%	17.742	3%	56.156	3%	82.223			3%	82.223	
2%	1%	11.035	3%	6.970	1%	19.010	3%	59.108	3%	92.814			3%	92.814	
	1%	12.038	3%	8.132	2%	20.277	1%	62.060	3%	103.405			3%	103.405	
	1%	13.042	1%	9.294	2%	21.544	1%	65.012	3%	113.996			3%	113.996	
	1%	14.045	1%	10.455	2%	22.811	1%	67.964	3%	124.587			3%	124.587	
	1%	15.048	1%	11.617	2%	24.078	1%	70.916	3%	135.178			3%	135.178	
2%	2%	16.051	1%	12.779	3%	1.373	1%	73.868	3%	145.769			3%	145.769	
	2%	17.054	1%	13.940	3%	2.746	1%	76.820	3%	156.360			3%	156.360	
	2%	18.057	1%	15.102	3%	4.119	1%	79.772	3%	166.951			3%	166.951	
	3%	1.056	1%	16.264	3%	5.492	1%	82.724	3%	177.542			3%	177.542	
	3%	2.112	1%	17.425	3%	6.865	2%	85.676	3%	188.133			3%	188.133	
2%	3%	3.168	2%	18.587	3%	8.237	2%	88.628	3%	198.724			3%	198.724	
	3%	4.224	2%	19.749	3%	9.610	2%	91.580	3%	209.315			3%	209.315	
	3%	5.280	2%	20.910	1%	10.983	2%	94.532	3%	219.906			3%	219.906	
	3%	6.336	2%	22.072	1%	12.356	3%	97.484	3%	230.497			3%	230.497	
	3%	7.392	2%	23.234	1%	13.730	3%	100.436	3%	241.088			3%	241.088	
2%	3%	8.448	2%	24.395	1%	15.103	3%	103.388	3%	251.679			3%	251.679	
	1%	9.504	3%	1.215	1%	16.475	4%	1.680	3%	262.270			3%	262.270	
	1%	10.560	3%	2.429	1%	17.848	3%	3.360	3%				3%		
	1%	11.616	3%	3.644	1%	19.221	3%	6.756	1%	16.052			1%	16.052	
	1%	12.672	3%	4.858	1%	20.594	3%	10.138	1%	20.086			1%	20.086	
2%	1%	13.728	3%	6.072	2%	21.967	1%	13.518	1%	24.079			1%	24.079	
	1%	14.784	3%	7.287	2%	24.712	1%	16.897	2%	28.093			2%	28.093	
	1%	15.840	3%	8.502	2%	27.458	1%	20.377	2%	32.105			2%	32.105	
	2%	16.896	3%	9.716	2%	30.204	1%	23.656	2%	36.118			2%	36.118	
	2%	17.952	1%	10.931	3%	32.950	2%	27.036	2%	40.131			2%	40.131	
2%	2%	19.008	1%	12.145	3%	1.479	2%	30.415	2%	44.144			2%	44.144	
	2%	20.064	1%	13.360	3%	2.957	2%	33.795	3%	48.157			3%	48.157	
	3%	1.109	1%	14.574	3%	4.426	2%	37.174	3%	52.170			3%	52.170	
	3%	2.218	1%	15.789	3%	5.914	3%	40.554	3%	56.184			3%	56.184	
	3%	3.327	1%	17.003	3%	7.393	3%	43.933	3%	60.197			3%	60.197	
2%	3%	4.436	1%	18.218	3%	8.871	3%	47.313	4%	64.210			4%	64.210	
	3%	5.545	2%	19.432	3%	10.350	3%	50.692	4%	68.223			4%	68.223	
	3%	6.654	2%	20.647	1%	11.828	4%	1.785	5%	72.255			5%	72.255	
	3%	7.763	2%	21.861	1%	13.307	4%	3.691		76.268				76.268	
	1%	8.872	2%	23.076	1%	14.786	3%	7.181		80.281				80.281	
2%	1%	9.981	2%	24.290	1%	16.264	3%	10.772	1%	16.897			1%	16.897	
	1%	11.090	2%	25.505	1%	17.742	1%	14.364	1%	21.121			1%	21.121	



Double Ducat.—\$4.40



10 Scudo —\$10



Sovereign.—\$6.50.



20 Lire.—\$3.80.



10 Lire.—\$1.90



5 Gilder.—\$2



Ducat.—\$2.20.



5 Thaler.—\$3.90.



80 Lire.—\$15.32.



5 Thaler.—\$3.90.



20 Lire.—\$3.80.



1/2 Joe.—\$1.75.



Sequin.—\$2.20.



Ducat.—\$2.20.



Ducat.—\$2.20.



20 Lire.—\$3.80



Ducat.—\$2.20

**GOLD COIN.—German and Ital.    SILVER COIN.—U. S.**



Quadruple Ducat.—\$8.80.



100 Lire—\$19.00



Gold Crown—\$5.72.



½ Imperial—\$3.90.



5 Roubles.—\$3.90.



10 Guilders—\$3.95.



20 Lire.—\$3.82.



2 Christian d'or \$7.80.

*U. States Silver Coins.*



5 Guilders—\$1.97.



U. States—One Dollar.



Hol. Ducat—\$2.15.



½ dime, 5 cents.



One Dime, 10 cts.



One Dollar, \$1.



One Dollar, \$1.



Half Dollar, 50 cents.



Half Dollar, 50 cents.



Half Dollar, 50 cents.



Half Dollar, 50 cents.



Quarter Dollar, 25 cents.

 $\frac{1}{4}$  Dollar, 25 cents. $\frac{1}{4}$  Dollar, 25 cents.



One Dollar, \$1.



One Dollar, \$1.



Dollar. \$1.00.



Crown—\$1.12.



$\frac{1}{4}$  Dollar. 25 cents.



Dime. 10 cents.



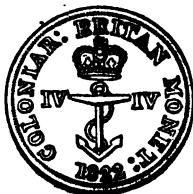
7 cents.



$\frac{1}{4}$  Crown, 12 cents.



Shilling—20 cents.



$\frac{1}{4}$  Dollar—24 cents.



Shilling, 20 cents.



Crown, \$1.12.



Crown—\$1.12.



5 Shillings. \$1.00.



Crown. \$1.05.



25 cents.



1 Shilling 18 cents.



Shilling. 20 cents.



1 Shilling. 18 cents.



18 cents.



1 Shilling. 18 cents



Crown. \$1.05.



Crown. \$1.05.



3 cent.



Crown—\$1.12.



5 cents.



3 Pence. 5 cents.



5 cents.



10 cents



1 Rupee. 40 cents.



1/8 Dollar. 10 cts.



11 cents



Sixpence, 10 cents.



5 cents.



10 cents.



One Dollar, 92 cents.



One Dollar, 92 cents.



Crown. \$1.05.



Half Crown, 56 cts.



50 cents.



Old Half Crown, 65 cts.

 $\frac{1}{8}$  Dollar. 10 cts. $\frac{1}{2}$  Pistareen. 8 cts.

Real. 10 cents.



10 cents.



$\frac{1}{2}$  Crown. 50 cents.



Pistareen. 16 cents.



One Dollar, \$1.



Dollar. \$1.



$\frac{1}{2}$  Dollar. 50 cents.



$\frac{1}{2}$  Dollar. 50 cents.



Head Pistareen, 18 cts.



50 Centimes. 10 cts



Pistareen, 16 cents.,



Pistareen, 16 cents.



100 Centimes. 20 cts.



Pistareen, 16 cents.



Scudo. 93 cents.



Scudo, of Naples. 94 cents.



$\frac{1}{4}$  Dollar. 25 cents.



Pistareen, 16 cts.



$\frac{1}{4}$  Dollar. 25 cents.



$\frac{1}{4}$  Dollar. 25 cents.



Gross Pistareen. 17 cts.



Pistareen. 16 cents



Dollar. \$1.00.



Ducat, of Naples. 94 cents



25 Centimes. 5 cts.



$\frac{1}{2}$  Dollar. 40 cents.



$\frac{1}{4}$  Patacon, 9 cts.



1 Real, 18 cents.



$\frac{1}{4}$  Dollar. 25 cents.



2 Reals, 25 cents.



$\frac{1}{4}$  Dollar. 20 cents.



1 Real, 18 cents.



Two Real, 18 cents.



1 Real, 18 cents.



1 Real, 18 cents.



Half Real 6 cts.



½ Real, 6 cents.



Half Dollar, 50 cents.



Quarter Dollar, 25 cts.



Dollar. \$1.00.



Half Dollar. 50 cents.



One Real, 11 cts.



Half Dollar. 45 cents.



Real. 10 cents



15 Sols, 12 cents.



¼ Lira, 4 cts.



40 cents,



1 Livre—17 cents.



10 Pauli of Tuscany. 98 cents.



$\frac{1}{4}$  Crown. 50 cents.



Crown. \$1.00



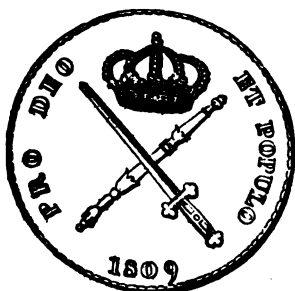
$3\frac{1}{2}$  Guilders. \$1.25.



$3\frac{1}{2}$  Guilders. \$1.25.



Crown Dollar [German.] \$1.00.



Crown of Bavaria. \$1.00.



\$1.20.



Crown. \$1.05.



Specie Rix Dollar. \$1.00.



3½ Guilders. \$1.25.



Denmark Crown. \$1.00.



2 Thalers. \$1.25.



Specie Rix Dollar. \$1.00.



3 Guilders. \$1.25.



$2\frac{1}{2}$  Thaler. 40 cents.



Bavarian Crown. \$1.00.



$2\frac{1}{2}$  Guilders. 90 cents.



36 Grotes. 25 cents. Real. 10 cents.  $\frac{1}{4}$  Crown. 25 cents.



$\frac{1}{2}$  Thaler. 20 cents. 16 cents. 5 Batzen. 8 cents.



5 Lire. 93 cents. Specie Rix Dollar. \$1.00.



24 Skillings. 20 cts.  $\frac{1}{4}$  Specie Rix Dol. 25 cts. 8 Skillings. 8 cts.



Specie Rix Dollar. \$1.00.



**Thaler. 60 cents.**



Rix Dollar. 60 cents.



24 Grochen. 40 cents.



**1 Franc. 15 cents.**



2 Francs. 30 cents.



1 Franc. 15 cents.



2 Livres. 30 cents.



30 Sols, 25 cents.



1 Livre. 15 cents.



1 Thaler. 60 cents.



1 Florin. 45 cents.



5 Francs. 93 cents.



5 Francs. 93 cents.



Five Francs, 93 cents.



Five Francs, 93 cts.



5 Livres 93 cents.



Five Francs, 93 cents.



2 Francs—34 cents



1 Thaler. 60 cents.



25 Centimes. 10 cts



1/2 Franc—8 cents.



6 Francs — \$1.05.



15 Sols—12 cents.



1/2 Livre. 8 cts.



10 Sous 8 cents.



Half Real, 6 cents.



Half Real, 6 cts.



2 cents.



2 cents.



7 cents.



One Dollar, \$1.



One Dollar, 90 cents.



One Dollar, \$1.



One Dollar, \$1.



Quarter Dollar, 22 cents.



Quarter Dollar, 22 cents.



Two Reals, 32 cents



Two Reals, 22 cents.



Quarter Dollar, 22 cents.



2 Reals, 16 cents.



Base Dollar. 65 cents



Dollar. 80 cents.



One Dollar, \$1.



One Dollar, \$1.



1 Real, 12 cents.



2 Reals, 22 cents



One Real, 12 cts.



Two Reals, 22 cents.



Quarter Dollar, 22 cents.



10 cents.



One Dollar. \$1.



One Dollar, \$1.



Dollar. \$1.00



Dollar. \$1.00



Half Dollar. 45 cents.



2 Reals, 16 cents



Two Reals, 20 cents.



1 Real, 6 cents.



One Real, 6 cts.



8 cents.



7 cents.



$\frac{1}{2}$  Dollar. 50 cents



$\frac{1}{4}$  Real. 3 cts.



Half Dollar, 35 cents.



Base Dollar. 70 cents



One Dollar, \$1.



One Real, 12 cts.



One Real, 11 cents



One Real, 9 cents



Quarter Real, 3 cts



Dollar. \$1.00.



Half Dollar, 50 cents.



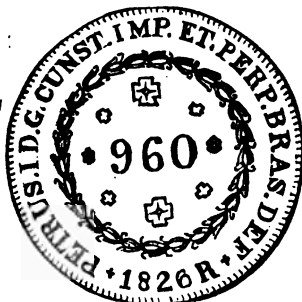
960 Reis. \$1,00.



960 Reis. \$1,00



Half Testoon, 4 cts. Real. 10 cents 100 Reis. 8 cents. Half Testoon, 4 cts



**980 Reis, \$1.**



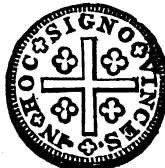
960 Reiz, \$1



**Florin—20 cents.**



[No Quotation.]



**Tartoon, 8 cents.**



1/4 Dollar. 25 cents.



150 Reis, 16 cents.



150 Reis, 16 cents.



Half Dollar. 45 cents



Half Dollar. 45 cents.



960 Reis [Brazil]. \$1.00



1200 Reis, \$1.



1 Real. 8 cents. 1/2 Rupee—20 cents. Half Real, 5 cents. Half Real, 6 cts.



Geneva Crown. \$1.00



1900 Reis, \$1



Five Pesetas, 90 cents.



960 Reis, \$1.



1/4 Rupee, 10 cents.



60 Reis, 6 cents.



Rouble. 75 cents.



60 Reis, 6 cents.



1/4 Rupee, 10 cts.

